PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 2 APRIL 15, 1916 Number 4

PRELIMINARY REPORT UPON THE POSSIBILITY OF CONTROLLING THE LAND SLIDES ADJACENT TO THE PANAMA CANAL

By the Committee of the National Academy of Sciences Appointed at the Request of the President of the United States

Dated, February 4, 1916. Received, March 16, 1916

INTRODUCTION

The Committee of the National Academy of Sciences, appointed November 18, 1915, at the request of President Woodrow Wilson "to consider and report upon the possibility of controlling the slides which are seriously interfering with the use of the Panama Canal," submits this its preliminary report.

The Committee as originally appointed consisted of thirteen persons. For various reasons four (Messrs. A. L. DAY, G. F. BECKER, C. D. WALCOTT, and R. S. WOODWARD) were unable to visit the canal and participate in the deliberations of the Committee. Those who took part in the preparation of this report are as follows:

C. R. VAN HISE,	WHITMAN CROSS,	J. R. FREEMAN,
H. L. ABBOTT,	R. C. CARPENTER,	J. F. HAYFORD,
J. C. BRANNER,	A. P. DAVIS,	H. F. REID.

These members, who will be spoken of as 'the Committee' in the report, sailed from New Orleans December 11, and arrived at Panama December 19. All spent two weeks in the Canal Zone, and three of them several days longer, working upon the problem submitted to them.

The part of the Canal cut between Bas Obispo and Pedro Miguel will be called the Gaillard Cut in accordance with the official use of that term. The deepest part of the Gaillard Cut, at the continental divide, about one mile in length, will be called the Culebra District.

The hill upon which the village of Culebra stands will be called Culebra Hill.

The general direction of the Canal is nearly northwest and southeast, but nearer north and south than east and west. The various stretches vary considerably in this direction. For the sake of brevity in description, the Canal will be regarded as running north and south, and directions at right angles to the Canal will be called east and west and those parallel with it north and south.

The term 'slides' when unqualified will be applied alike to material which is now in motion and to that which once has been in motion but is now quiescent. Where it is necessary to discriminate between these two conditions of the slides, one will be called 'active' and the other 'quiescent.'

The Culebra district was visited by all members of the Committee five days, and a number of the Committee spent several additional days in this area. The attention of the Committee was primarily directed to the question of the control of the active East and West Culebra Slides, but the extensive Cucaracha Slide, now quiescent, was also examined with care. The Committee also examined the massive hills of the Culebra District and especially Gold, Culebra, Zion and Contractors Hills, all of which are adjacent to the great slides.

The work of the Committee in the field was facilitated in every way by Major General George W. Goethals, Lieut. Col. Chester H. Harding, Lieut. Col. Jay J. Morrow and Rear Admiral H. H. Rousseau; General Goethals furnished records from his office, a brief history of the slides and their movements, and much other information which the Committee desired; in short, all possible help has been given to the Committee so so that its members could carry on their work most effectively in their own way.

The Committee has profited greatly by the geological studies of Mr. Donald F. MacDonald and by conferences with him in the field and in the office.

As the uninterrupted operation of the Panama Canal is a matter of great national importance, the Committee plans a further study of the available data and expects in due time to make a fuller report; but it seems desirable to present promptly to President Wilson, a preliminary report containing the views of the Committee (so far as they can now be formulated), and such practical suggestions as the Committee is able to offer.

THE THREE GREAT SLIDES

The slides which led to the closing of the Canal on September 18, 1915, were the great East Culebra and West Culebra Slides. General Goethals has described these slides in an article prepared for the press, under date November 15, 1915, and from this article the following statements are taken:

The East Culebra Slide began on October 14, 1914, without any warning and a section of the east bank north of Gold Hill settled vertically 20 feet. This section measured 2,000 feet (now extended to 2,700 feet) along the prism face and extended back 1,000 feet from the axis of the Canal, generally along an irregular curved line. The top of the bank was from 300 to 350 feet above sea level, and the extension of the ground eastward was relatively flat country. In the settlement, the upper portion which broke away remained practically parallel to its original position, and the benches which formed the upper part of the slide had not changed their relative positions, though they were badly broken up, while the lower strata were squeezed out across the Canal. Subsequently the broken mass moved into the cut, reducing the depth of the water from 45 feet to 9 inches at one point. Until August, 1915, the dredges were able for the most part to keep up with the movement as it came down, and probably would have been able to maintain this condition had not a movement occurred on the west bank, necessitating work on this side to the detriment of the east side.

A crack was found on the slope of Zion Hill in June, 1914, but observations made upon it showed no movement and the solidity of the hill was never doubted. Subsequent to the break on the east side, a gradual but general breaking up of the west bank followed, and the crack on the slope increased in size and new ones developed farther up the hill, until finally one extended to the elevation of 480, the limit of the present break. The movement into the cut from the west bank occurred early in August, 1915, when a section of Zion Hill broke away and settled down. The edge of the break on this side is also a curve.

The movements from the two sides are towards the central portion of the enclosed area, and at this central portion is the obstruction to the channel. It first appeared as an island forced up from the bottom, then as a peninsula projecting from the east bank, and finally was pushed entirely across the channel completely closing it. . . . .

The length of the slides, which are directly opposite each other, is approximately 2,200 feet (the channel through which is navigable with the exception of 600 feet); the banks are 300 to 350 feet above sea level on the east and extend up to 480 feet above sea level on the west. The area of the territory affected on the east side covers 81 acres and on the west 78.5 acres.

Assuming that all material lying above planes extending from the outside limits of the bottom of the prism, reference 40, up to the limits of the

breaks, will move into the cut, 7,000,000 cubic yards will have to be removed before the slides are entirely stopped. Mr. Comber, Resident Engineer of the Dredging Division, assumed a surface parallel to the surface existing on October 14, the date of the last complete survey, and 45 feet below it, on which basis 13,000,000 cubic yards would be the quantity to be handled. He thinks, however, that a mean between the two amounts may be more nearly correct, which was the method of arriving at the 10,000,000 cubic yard figure which has appeared in the press. It is at best only a guess. It must not be inferred from this that the Canal will be closed until this amount is dredged, for such is not the case; on the contrary it is the intention to pass ships as soon as the channel is secured through the remaining 600 feet, and there are reasonable grounds for assuming that a channel through the obstructed area can be maintained.

The active West Culebra Slide extends 2900 feet along the Canal, and 1350 feet at right angles to it measured from the axis of the Canal to the furthest point of the slide.

Of the slides now quiescent, the most important is the Cucaracha. General Goethals writes of it, in the article already mentioned:

> On January 20, 1913, a break occurred at Cucaracha by which the rock bluff which was holding back the upper mass of clay broke at or below the bottom level of the Canal, completely filling the prism with clay and rock, reaching to 69 feet above sea level on the opposite or west side of the cut. The length of the prism so filled was 1600 feet. Steam shovels were scarcely able to keep pace with the movement, tracks were covered and disarranged, shovels overturned, and the difficulties of transportation increased, since only tail tracks sufficient for two or three cars could be maintained. Furthermore, the soft material increased the difficulties of the dumps. As the movement continued the clay broke farther and farther up the hillside.

The Cucaracha Slide, as stated, extended 1600 feet along the Canal, and 1880 feet at right angles to it.

Dominant Importance of the Three Great Slides.—According to Mr. W. G. Comber, Resident Engineer in charge of Dredging, the acreage of the three great slides is as follows:

West Culebra Slide	60.8
East Culebra Slide	70.5
Cucaracha Slide	60.4
Total	201.7

In contrast with this, the total area of all other slides is 112 acres.

The dominance of the three great slides appears even more marked when the amounts of excavated and moving material are considered.

Mr. Comber's figures for excavation accomplished to December 30, 1915, are:

	cu. yds.
West Culebra Slide	10,931,862
East Culebra Slide	14,687,563
Cucaracha Slide	9,901,602
Total	35,521,027

In contrast with this amount, the total excavations for the other slides to the same date have been 4,852,648 cubic yards.

If the amounts of material still to be removed are compared, the dominance of the three great slides is maintained. It is estimated that, on December 30, 1915, there remained to be excavated:

	cu. yds.
West Culebra Slide	3,500,000
East Culebra Slide	5,600,000
Cucaracha Slide	500,000
Total	9,600,000

The total amount still to be removed from all other slides is estimated at the insignificant amount of 330,000 cubic yards. It thus appears that the amount of material already excavated and still to be removed from the three great slides is estimated at 45,121,027 cubic yards; whereas, the corresponding amount for all the other slides is 5,182,648 cubic yards, or about one-ninth as much.

The foregoing facts are conclusive as to the dominant importance of East Culebra, West Culebra and Cucaracha Slides.

Since the three great slides are all in the Culebra District, this is the chief area of danger. This is the natural consequence of the fact that by far the deepest part of the Canal cut is in this District; and that the weakest of the geological formations, the Cucaracha, is strongly developed here.

Slides may occur in other parts of the Canal, but they will be relatively small, and infrequent; for the banks are not high, and the unstable ground has already slid down so that its surface has approached the angle of repose. Should slides occur, they are not likely to menace the operation of the Canal. Slides, great enough seriously to obstruct traffic in the Canal, could occur only in the Culebra District, which is but one mile long. The possibility of great slides in this section has therefore claimed the most careful consideration of the Committee.

General Features of the Culebra District.—In order that the views of the Committee may be clearly conveyed, it is necessary briefly to mention and to illustrate by a map and photographs, the more important features of the Culebra District.

The Canal here traverses the highest land in its course. On the east side, rising abruptly from the Canal, is Gold Hill. This is the highest hill near the Canal; it reaches a height of 660 feet above sea level, or 620 feet above the bottom of the Canal. It is composed mainly of hard intrusive basalt and hard tuff nearly surrounded by basalt. It separates the Cucaracha slide on the south from the East Culebra slide on the north. These slides have caused great breaks in the north and south flanks of Gold Hill, leaving nearly vertical cliffs, which are 275 feet high on the south flank. On the opposite western side of the Canal there are three prominent hills, Culebra, Zion, and Contractors Hills, in order from south to north. Their elevations are:

	Above Sea Level	*Above bottom of Canal*
Contractors Hill	415	375
Zion Hill	570	530
Culebra Hill	390	350

Zion Hill is of intrusive basalt, the other two mainly of hard tuff with some basalt. Contractors Hill nearly touches the waters of the Canal opposite Cucaracha Slide. Zion and Culebra Hills stand about 1500 and 1200 feet, respectively, from the Canal, and in front of them lies the West Culebra Slide. The slide has caused breaks in both of these hills. In the eastern front of Culebra Hill a road was carried down and a number of houses had to be removed; cracks roughly parallel to the Canal have formed for a distance of about 100 feet beyond the prominent break which seems to be the western limit of the slide at the present time, and extend to within 250 feet of the summit of the hill. Zion Hill also has suffered; a large mass has fallen from its eastern face leaving a vertical cliff, whose edge is only about 50 feet from the top of the hill. Contractors Hill has not been affected by the slides.

The great slides occur in the low ground adjacent to these hills where a large amount of softer rocks had already been removed by natural erosion before the excavation of the Canal was begun.

Relations of the Great Slides to the Hills.—The Cucaracha Slide is mainly confined to the area between Gold Hill and a subordinate basalt mass to the south. It extends from the Canal for a considerable distance east of the crest of Gold Hill; and its head reaches the subordinate

divide to the east. It is estimated that the Cucaracha Slide drains an area of 80 acres. The slide is sharply limited on the north by the break in Gold Hill already mentioned, which extends approximately at right angles to the Canal; its southern limit is not so well defined.

The limit of the East Culebra Slide is sharply marked on the south by the break through the north part of Gold Hill which runs approximately at right angles to the Canal. The Northern boundary is not so sharply marked. The slide extends slightly beyond the subordinate divide on the east, so that east of the slide the drainage is away from the Canal.

The West Culebra Slide is limited on the south by breaks beginning at the Canal some distance north of Contractors Hill; on the west by breaks which are sharply defined in Zion Hill; and less sharply in Culebra Hill. In the narrow valleys between Culebra and Zion Hills and between Zion and Contractors Hills the break extends beyond the divide, and the drainage is to the west.

Possible Extensions of the Great Slides.—The very important question now arises: Will the great slides extend their limits and cause further serious trouble?

In general, the Committee believes that no great extension of these slides is probable, because the soft rock constituting a very large part of the slides is quite limited in extent, except east of the East Culebra Slide, and conditions elsewhere are unfavorable for extensions.

The Cucaracha Slide cannot greatly extend its area on account of the basaltic intrusions which surround it; but its eastern and southern limiting banks are still breaking down, and the movement of the slide may be revived to a small degree. Plugs or branches of intrusive basalt standing as obstructions across the former course of the slide restrain its movement; but the strength of these obstructions cannot be determined from present exposures.

The West Culebra Slide is pretty definitely limited on the west by the hard rocks of Culebra and Zion Hills; but between the active part of the slide and Contractors Hill there is a considerable mass of the Cucaracha formation, which seems never to have taken part in the slides. The effect on this mass of the settling of the adjacent moving material cannot certainly be predicted. Indeed it is not impossible that a considerable part of it may finally be set in motion; but the mass involved will be small in comparison with the active West Culebra Slide.

East of East Culebra Slide the soft formation continues, but the slope is gently away from the Canal. Additions to the slide to the east

are possible, but because of the slope and increased distance from the Canal such possible additions would be in decreasing volume. Gold Hill limits the slide to the south.

THE SOLIDITY OF THE HILLS OF CULEBRA DISTRICT

In addition to the danger of the slides, is there danger that the Canal may be blocked by the fall of the hills of Culebra District?

These hills are composed of intrusive bodies of basalt, or of masses of the hard Obispo tuff commonly associated with basalt in this district. So far as can be judged from present exposures, they do not rest on the soft Cucaracha formation, but extend far into the earth, and are self-supporting. Rock may slough off from them, but there is no evidence that they will collapse.

Culebra and Zion Hills.—The hard tuff of Culebra Hill practically limits the West Culebra Slide in front of it. Cracks have formed in the tuff and it is probable that some of the rock will break off as the slide settles. Zion Hill is a basaltic intrusion, and much rock has fallen from its face; more may follow. But the total amount that may fall will only make a relatively small addition to the upper part of the West Culebra Slide.

Gold and Contractors Hills.—Gold and Contractors Hills rise steeply from the banks of the Canal, for 410 feet and 260 feet above the bottom of the Canal, respectively; and then slope more gently to their summits. They are nearly, but not exactly, opposite each other. Gold Hill is chiefly composed of basalt, which formerly spread out near its top, and was partially supported on the softer Cucaracha formation. When the East Culebra and Cucaracha slides became active the support was removed, and a large mass of the basalt fell from the northern and southern sides of the hill. The lower part of Gold Hill on the side towards the Canal is made up of hard Obispo tuff, bounded by a basalt dike, and there is little danger that it will yield.

Contractors Hill is of hard Obispo tuff, which is separated from the Cucaracha by a fault which dips into the hill at an angle of 60 or 70 degrees with the horizontal. There is a possibility that this part of the hill depends more for its support on the Cucaracha beds than seems probable, and as a measure of precaution all reasonable means should be taken to keep the Cucaracha beds in place; and, especially, the fault fissure should be kept closed to prevent water seeping in. If the borings, suggested later, to reveal the underground structure, show that these precautions are unnecessary they can be discontinued.

The excavation of the Canal, and borings in its bottom show that a narrow belt between the two hills is composed of the soft Cucaracha beds; yet to the present time there has been no upheaval of the bottom of this part of the Canal, nor any other sign to indicate that the hills have settled. It is believed, therefore, that the great masses of Gold and Contractors Hills are self-supporting and will remain so. There is no occasion to raze them.

CAUSES OF THE SLIDES

On account of their magnitude, the land slides have received serious consideration since the early days of the Canal. But before measures for their control are taken up it is necessary briefly to discuss their causes.

The slides in the Canal Zone are essentially like many in other parts of the world; they are due to the inability of the earth or rock to support the weight of overlying material. Slow processes of natural erosion, rapid cutting by flooded streams and excavations by man frequently lead to landslides. Much of the Canal is cut through weak rocks; and in the Culebra District the prism is exceptionally deep. It is clear that the conditions there are very favorable for slides.

The weakness of the rocks is due to several causes:

Character of the Rocks of the Culebra District.—The rocks of Culebra District are of two kinds—stratified and massive. The chief material involved in the slides is the stratified Cucaracha formation. It is greenish grey in color, largely composed of clayey material with some layers of rather finely banded volcanic sandstone or tuff, only weakly consolidated. The Cucaracha beds are limited, along the line of the Canal, to the Culebra District but they have a thickness in places of over 400 feet. The soft slippery nature of its materials and their loose, unconsolidated condition, make it unusually weak and unable to sustain any considerable load.

The Cucaracha beds alone are responsible for the great slides. The Obispo tuff is a rather coarsely fragmental rock, roughly stratified; masses of the tuff and of the massive columnar basalt have broken from the hills and added some material to the slides, but they have had no part in starting them.

Structural Weakness.—The rocks, both stratified and massive, as shown by the work of Mr. MacDonald, are cut by numerous faults and this is true of the rocks throughout the Gaillard Cut. Where there is a fault, the rocks have previously been broken; and therefore, present a place

of exceptional weakness. Some of the smaller slides outside the Culebra District are limited by faults.

An important source of weakness is the fracturing of the rocks by complex sets of joints. They occur in both stratified and massive rocks. The friable parts of the Cucaracha formation are broken by joints into fragments of various sizes down to a fraction of an inch. The massive rocks are fractured in every direction by joints on a much larger scale.

Earthquakes.—A consideration of the earthquakes felt in the Canal Zone and a careful examination of the instrumental records kept near the Pacific end of the Canal since the end of 1908 dispel fear of serious damage to the Canal or its accessories by earthquakes. A number of pretty sharp shocks have been felt in the Zone but they have originated at a distance of about 120 miles from the Zone. A few fairly sharp shocks had their origin about 80 miles distant; and two or three, which were not felt, and which made a very feeble record on the delicate seismographs, were much nearer. At the time of the very sharp shocks of October, 1913, a prism of earth 60 feet high and with a base of about 100 square feet broke off the steep slope near the top of the Cucaracha Slide; this is quite insignificant. There is no evidence that any of the slides have been started or increased by earthquakes.

The Heavy Rainfall.—Investigations in many parts of the world have shown that excessive water in the ground is a great promoter of landslides. The strength of the Cucaracha clays is greatly decreased by the presence of water, and the heavy tropical rains keep them nearly saturated the greater part of the year; for the broken and irregular surface of the slides and the open cracks around their borders greedily drink in the water that falls upon them.

In the Culebra District the average rainfall since 1884 has been 87.68 inches per annum, and during the time of the excavation of the Canal it has been 84.75 inches per annum. Moreover this rainfall is almost wholly concentrated in eight months of the year. The average for the rainy months during the period of excavation of the Canal by the United States, from the middle of April to the middle of December has been 80.01 inches. Where nearly seven feet of water fall upon the surface of the country within eight months of the year, it is not surprising that there is difficulty in controlling the underground seepage. Indeed during these months the ground water table is practically at the surface, except possibly on the steeper slopes; and the movement of the underground water is so slow that even in the dry season the water table is not far from the surface in the level country. The New French Company found that the water table at the two points of the East Culebra

Slide was six and sixteen feet below the surface respectively. On the summit of Gold Hill, the ground water falls 40 feet below the surface in the dry season, as indicated by the zone of weathering.

REMEDIAL MEASURES

All slide material which reaches the Canal must, of course, be removed at a certain expense and inconvenience. This work is being prosecuted by the engineers with great vigor. They estimate that about 9,000,000 cubic yards will have to be removed between the two Culebra Slides; and the dredges can remove 1,000,000 cubic yards a month. But it must not be supposed that the Canal will remain closed for nine months; as soon as the channel has been sufficiently deepened and the movement of the slides becomes so slow that the dredges, even when interrupted by the passage of ships, can more than keep pace with them the Canal may be opened for navigation. This time is probably not far off.

The Committee believes that some sliding ground will continue to enter the Canal for several years to come, though in diminishing amounts. Any relatively inexpensive measures which tend to arrest the present active slides, or which promise to reduce the charge against maintenance of the Canal in the future are fully warranted.

The composition of the rocks, their structural weakness, and earthquakes are beyond the control of man, but a partial control of ground and rain water and the relief of pressure by unloading certain areas are feasible, and the Committee will confine its suggestions to these measures.

Control of the Water.—As early as the time of the first French Company the advantage of controlling the water was recognized; and the New French Company made several attempts to keep the water out of the relatively small slides of their time by surface drains and tunnels, but with only partial success. The Committee believes that every available and practicable device should be used to turn the water falling as rain from all ground that is sliding and prevent its entering adjacent ground, and it suggests the following measures:

1. Covering Slopes with Vegetation.—Whether vegetation increases or decreases the amount of rainfall entering the ground is still a moot question. The Committee believes, however, that threatening ground bordering the slides, quiescent slides, and, so far as practicable, active slides themselves, should be sufficiently covered with vegetation to prevent surface wash.

2. Closing Peripheral Cracks.—Before extensive movements of the

ground occur, warning is frequently given by the appearance of cracks peripheral to the coming slide. In some instances cracks exist for a long time before important movements take place. They are well illustrated upon Culebra Hill, west of the great break. As soon as they are formed they should be filled up, in order that they may not intercept surface water and lead it into the slide.

3. Drainage of Undisturbed and Threatened Areas.–Undisturbed and threatened areas near the slides should be thoroughly drained both by surface and by tile drainage, to keep as much water out of them as possible, for they may become unstable and they may supply water to the slides. The drainage water should be carried from the neighborhood of the slides as directly as practicable. Experiments with tile drainage on a small scale would soon show whether it is effective enough to justify its extension. The surfaces of the ground east of the East Culebra Slide and in places west of the West Culebra Slide, slope away from the adjoining slides, and the water can be readily removed. But a considerable area above the Cucaracha Slide drains naturally into it; some of the drainage can be diverted to the east, but the rest should, so far as possible, be collected by surface and tile drains, and be carried to the Canal through a large concrete-lined surface drain.

4. Drainage of the Great Slides.—A complete system of open drains should be established on the great slides and the water carried away as directly as possible. The main drains should be made impervious on the Cucaracha and, so far as practicable, on the two Culebra Slides.

5. Drainage by Tunnels.—Drainage by means of tunnels might be adapted to a few special cases, but should be tried with caution, and extended only in cases which promise results commensurate with the cost. The tunnels should be built underneath the sliding ground in the undisturbed material and strongly timbered so as to avoid risk of collapse, which would not only destroy the tunnel but would also disturb the overlying material. From the main tunnel smaller branches may be extended into the material to be drained and frequent borings made from the surface to conduct drainage water to the tunnels from the overlying strata. Such a system would be expensive, but might be effective in draining the area tapped by it.

Relief of Pressure.—It is suggested that a cut be made by sluicing in the East Culebra Slide, starting at an appropriate point on the Canal, and diverting from it diagonally in a general southeasterly direction, in the zone of gentle slopes and in such a position as to reach the large pond which now exists on the slide. This cut would partly separate the roughly rectangular slide into two approximately triangular parts, and

the adjoining ground could be sluiced along it into the Canal. It would provide a main drainage line for the ground on both sides of it, would empty the surface ponds and would lower the ground water in adjacent sliding material. The pressure of the triangle of ground in the rear of the cut against the triangle in front of it would be lessened; and it is probable that movement in the forward part of the slide would cease sooner than it would otherwise. When the cut is once established it offers a second line of defense against the slide by making it possible to work along two fronts.

A somewhat similar cut should be made in the West Culebra Slide. It should begin at the main drainage line opposite Culebra Hill and extend diagonally from the Canal in a general southerly direction, and should drain the existing pond.

STUDIES FOR THE FUTURE

In addition to the immediate remedial measures suggested above, there are certain observations and protective measures which should be undertaken with a view to the future.

Detection of Movements of Earth or Rock.—A few slides are now in motion, but many more are quiescent. A slide is not necessarily 'dead' because it has not moved notably for a few years. Its stability may hang on a delicate balance which may be disturbed by some slowly developing weakness. There is also a bare possibility that Gold and Contractors Hills are not so firmly supported as they are believed to be. Repeated surveys of properly placed signals for a number of years to come should be made in order:

(a) To indicate in good time where additional work of prevention is needed, to indicate how large a dredging fleet must be kept in readiness, and to furnish to the engineer of maintenance advance indications of emergency conditions.

(b) To test the effectiveness of remedial measures which may be taken to control or prevent slides. These tests would be more sensitive and definite than the mere occurrence or non-occurrence of slides.

(c) To furnish a reliable basis for confidence when, in the course of time, it appears that the earth and rock movements have so decreased that a condition of stability may be inferred.

Core Borings.—Definite information relative to the rock underlying Gold and Contractors Hills is wanting. Many years ago some borings were made at stations 500 feet apart along the center line of the Canal, some of them to a depth of 40 feet below sea level.

Two or three core borings should be made on each side of the Canal with a drill which would give large cores. At least one horizontal and one inclined hole should be bored on each side, their precise location to be fixed by a geologist.

Underground Water and Related Data.—As underground water is of paramount importance in promoting slides, it should be carefully studied in the Canal Zone. A satisfactory investigation of this problem would require the following determination and studies:

1. Profiles of water table for different localities of the Culebra District:
   (a) In the wet and dry seasons;
   (b) In areas in which remedial measures have been applied and similar areas where they have not.
2. Percentage of porosity of the several formations of the Culebra District:
   (a) Absolute;
   (b) With regard to size of grain.
3. The effect of tropical vegetation with the accompanying humus, its removal and its restoration, on:
   (a) The amount of water which sinks underground in sliding and in undisturbed areas;
   (b) Chemical action, such as oxidation, hydration, carbonation, etc.;
   (c) The composition of the water;
   (d) Changes in underground temperatures.
4. The nature of the changes which result in the disintegration and decomposition of the various rocks of the Culebra District when exposed to weathering agencies:
   (a) With regard to volume;
   (b) With regard to chemical changes, i.e., oxidation, hydration, carbonation, action of acids, etc.
5. The chemical changes which cause the so-called hot areas described by Colonel Gaillard and Mr. MacDonald:
   (a) Direct, in production of acids, etc.;
   (b) Indirect, i.e., the effect of produced acids upon the materials, including water and rock.
6. Observations to determine whether similar changes to those in the hot areas take place elsewhere to a less extent over large areas, and whether such changes affect the slides.

Mechanical Testing of the Rocks.—Sliding is largely dependent upon the strength of the rocks; which in turn is affected by the water content. The strength of the massive igneous rocks is well known to be great,

and need not be determined. But the Obispo, Cucaracha, and other formations of the Gaillard Cut should have their strength tested when saturated with water, when moist and when dry; and under rapid and slow deformation. If the tests show that much less force is required to deform the rocks when saturated or moist than when dry, this will emphasize the importance of keeping the water from these rocks so far as possible.

The tests should be made on fresh rocks and therefore in the Canal Zone. The specimens tested should be as large as is feasible for a testing machine of 200,000 pounds capacity.

Earthquake Studies.—There are now two seismographs installed in the Administration Building at Balboa Heights. It would be an advantage if the smaller instrument should be removed to a second station, for instance Colon, in order that the origin of earthquakes, occurring in regions within two or three hundred miles of the Canal Zone, may be more definitely determined. Some of the stronger shocks felt in the Zone have thrown the needles of the delicate seismographs off the paper and left the records incomplete. A low power instrument, magnifying about four times would secure a record of the movements of the ground in these cases.

GENERAL CONCLUSIONS

It is obvious that the sliding material which enters the Canal must be removed. The important thing for the future is to prevent ground from entering the Canal. The chief remedy proposed by the Committee to retard the movement of the slides now in motion and to prevent the slides from extending their areas, is to reduce the amount of water which goes underground. Methods have been suggested by which this can be done; and they should be vigorously applied to all moving and threatening areas.

The Committee looks to the future of the Canal with confidence. It is not unmindful of the labor necessary to deal with the present slides; and it realizes that slides may be a considerable, but not an unreasonably large, maintenance charge upon the Canal for a number of years; it also realizes that trouble in the Culebra District may possibly again close the Canal. Nevertheless, the Committee firmly believes that, after the present difficulties have been overcome, navigation through the Canal is not likely again to be seriously interrupted. There is absolutely no justification for the statement that traffic will be repeatedly interrupted during long periods for years to come. The Canal will serve the great purpose for which it was constructed, and the realization of that purpose in the near future is assured.

DISCOVERY OF EIGHT VARIABLE STELLAR SPECTRA

By Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, February 23, 1916

In three earlier communications to the PROCEEDINGS various phases of the problem of Cepheid variation have been considered.[1] No further introduction to the present note is necessary beyond the statement that the periodic change in color, which appears to be typical of all Cepheids, has already been found to correspond to normal changes of spectral class for two cluster-type stars, RS Boötis and RR Lyrae, and for one variable of longer period, δ Cephei. In order to test to what extent the inconstancy of spectrum is a general phenomenon of Cepheids, some 150 spectrograms of representative variables of this class have been made with the 10-inch portrait lens and objective prism. The periods of the stars investigated range from 9 hours to 27 days; many are those for which spectroscopic orbits have been computed; some are well-known naked eye variables, others are much fainter stars; for some, changes in color have been suspected from studies of the light curves, for others the maximum intensity of the spectrum has been observed to shift toward the blue upon the approach to maximum light. For none, however, has it been suggested, so far as I know, that the spectrum changes periodically along the normal spectral series.

Following the numerical method of classification recently described by Adams,[2] the change of spectrum of Cepheids is susceptible of easy detection, even when the variation is very small. For example, in the work with low dispersion on F-type spectra the relative intensity of the *G* band and the hydrogen line $H\gamma$ is particularly capable of showing small changes of type. The results can be summarized briefly.

The table contains data relative to the spectra of eleven stars (all that are now known to have variable spectra), and some information concerning their light fluctuations. The observed range of spectrum variation is in nearly every case smaller than the probable total range, as observations at the exact time of maximum and minimum were not made. The accompanying figure contains two objective-prism spectra of δ Cephei, showing the conspicuous change, from minimum to maximum light, in the relative intensity of the spectral lines. The more detailed appearance with high dispersion of a

FIG 1. SPECTRA OF δ CEPHEI NEAR MAXIMUM (ABOVE) AND MINIMUM OF LIGHT (enlarged five times).

Eleven Cepheids with Variable Spectra

STAR	MAXIMUM MAGNITUDE	RANGE OF VARIATION	PERIOD	NUMBER OF SPECTROGRAMS	OBSERVED SPECTRUM VARIATION
		mag.	*days*		
TU Cassiopeiae	7.3	1.1	2.139	9	F0 to F6
SU Cassiopeiae	5.9	0.4	1.950	19	A8 to F5
SZ Tauri	7.2	0.5	3.148	11	F4 to G2
T Monocerotis	6.0	0.8	27.012	6	F4 to F8
RT Aurigae	5.0	0.9	3.728	12	A8 to G0
W Geminorum	6.4	1.3	7.916	10	F3 to G0
RS Boötis	8.9	1.1	0.377	13	B8 to F0
X Sagittarii	4.4	0.6	7.012	5	F2 to G
Y Ophiuchi	6.2	0.8	17.121	4	F5 to G0
RR Lyrae	6.8	0.9	0.567	17	B9 to F2
δ Cephei	3.5	0.8	5.366	21	F2 to G3

smaller section of the spectrum has been shown in an earlier communication by Adams and Shapley.

Every variable for which the present test is sufficient was found to vary in spectrum. It appears safe to infer, therefore, that all Cepheids (including the cluster-type), besides being variable in light and in velocity, vary periodically in spectral class as well.

[1] Shapley and Shapley, these PROCEEDINGS, 1, 452 (1915); Shapley, *Ibid.*, 2, 132 (1916) Adams and Shapley, *Ibid.*, 2, 136 (1916).

[2] These PROCEEDINGS, 2, 143 (1916).

ON THE LINEAR DEPENDENCE OF FUNCTIONS OF SEVERAL VARIABLES, AND CERTAIN COMPLETELY INTEGRABLE SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

By Gabriel M. Green

DEPARTMENT OF MATHEMATICS, HARVARD UNIVERSITY

Received by the Academy, February 25, 1916

The study of an ordinary homogeneous linear differential equation of the nth order leads very naturally to the definition of the Wronskian of n solutions of the equation, and thence to the general theory of the linear dependence of n functions of a single variable. This is due to the characteristic property of the said differential equation, viz., that any solution of the equation is linearly dependent upon any fundamental set of solutions. I wish in this note to give some of the results which I have obtained in generalizing the theory of linear dependence to the case of n functions of several independent variables, and also to point out the application of these results to the study of an important

class of systems of partial differential equations which are a direct generalization of the single ordinary homogenous linear differential equation of the nth order. A system of the kind referred to contains a single dependent variable and any number of independent variables, and has a fundamental system of solutions, that is a definite number of linearly independent solutions in terms of which any other solutions of the system of differential equations is expressible linearly, with constant coefficients.

The following discussion is concerned throughout with functions of p independent variables. If any of the variables be complex, we shall suppose the functions to be analytic in those variables. However, we shall state all theorems for the case in which the independent variables are real, and the functions either real or complex; the modifications which must be made if some or all of the variables be complex are easily supplied, and will need no further mention. We shall impose upon the functions no restriction other than the existence of certain partial derivatives in a certain connected p-dimensional region A of the space of the independent variables.

Let $y_1, y_2, \ldots, y_n$ be functions of the p independent variables $u_1, u_2, \ldots, u_p$. We shall denote by $y_i^{(1)}, y_i^{(2)}$, etc., partial derivatives of y_i, of any kind or order whatever. It will be unnecessary to specify just what derivative of y_i is denoted by $y_i^{(j)}$. However, in any given discussion the same superscript (j) will denote the same derivative throughout. If a derivative $y^{(j)}$ exists for each one of the set of functions y_i $(i = 1, 2, \ldots, n)$, we shall say that *the set of functions possesses that derivative.* We may now state the fundamental theorem concerning the linear dependence of functions of several variables:

THEOREM I. *Let the set of n functions $y_1, y_2, \ldots, y_n$ of the p independent variables $u_1, u_2, \ldots, u_p$ possess enough partial derivatives, of any orders whatever, to form a matrix.*

$$M \equiv \begin{Vmatrix} y_1 & , & y_2 & , \cdots , & y_n \\ y_1^{(1)} & , & y_2^{(1)} & , \cdots , & y_n^{(1)} \\ y_1^{(2)} & , & y_2^{(2)} & , \cdots , & y_n^{(2)} \\ \cdot & & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot \\ y_1^{(n-2)} & , & y_2^{(n-2)} & , \cdots , & y_n^{(n-2)} \end{Vmatrix}$$

of $n-1$ rows and n columns, in which at least one of the $(n-1)$-rowed determinants, say

$$W_n \equiv \begin{vmatrix} y_1 & , & y_2 & , & \cdot & \cdot & \cdot & , & y_{n-1} \\ y_1^{(1)} & , & y_2^{(1)} & , & \cdot & \cdot & \cdot & , & y_{n-1}^{(1)} \\ \cdot & & \cdot & & & \cdot & & & \cdot \\ \cdot & & \cdot & & & \cdot & & & \cdot \\ y_1^{(n-2)} & , & y_2^{(n-2)} & , & \cdot & \cdot & \cdot & , & y_{n-1}^{(n-2)} \end{vmatrix},$$

vanishes nowhere in A. Suppose, further, that all of the first derivatives of each of the elements of the above matrix M exist, and adjoin to the matrix M such of these derivatives as do not already appear in M, to form the new matrix

$$M' \equiv \left\| \begin{matrix} y_1 & , & y_2 & , & \cdot & \cdot & \cdot & , & y_n \\ y_1^{(1)} & , & y_2^{(1)} & , & \cdot & \cdot & \cdot & , & y_n^{(1)} \\ \cdot & & \cdot & & & \cdot & & & \cdot \\ \cdot & & \cdot & & & \cdot & & & \cdot \\ y_1^{(n-2)} & , & y_2^{(n-2)} & , & \cdot & \cdot & \cdot & , & y_n^{(n-2)} \\ \cdot & & \cdot & & & \cdot & & & \cdot \\ \cdot & & \cdot & & & \cdot & & & \cdot \\ y_1^{(q)} & , & y_2^{(q)} & , & \cdot & \cdot & \cdot & , & y_n^{(q)} \end{matrix} \right\|$$

which has n columns and at least n rows, so that $q \geqq n - 1$. *Then if all the n-rowed determinants of the matrix M′ in which the determinant* W_n *is a first minor vanish identically in A, the functions* $y_1, y_2, \ldots, y_n$ *are linearly dependent in A, and in fact*

$$y_n = c_1 y_1 + c_2 y_2 + \ldots + c_{n-1} y_{n-1},$$

the c's being constants.

The proof of this theorem is very similar to the familiar one of Frobenius for functions of a single variable [Cf. M. Bôcher, *Trans. Amer. Math. Soc.*, 2, 139–149 (1901)]. For $p = 1$, the theorem becomes a generalization of the ordinary Wronskian theorem for functions of a single variable, and includes the latter theorem as a special case.

It should be noted that for functions of several variables it is not possible to define a single determinant which may properly be called a Wronskian; however, a Wronskian may be defined for a completely integrable system of partial differential equations, of the kind mentioned above. Before giving this definition it will be convenient to state an existence theorem for the system of partial differential equations.

Let us call a set of derivatives $y, y^{(1)}, y^{(2)}, \ldots, y^{(n-1)}$ of a function y a *normal set*, if for every element $y^{(r)}$ of the set there exists in the set at least one other element $y^{(q)}$, whose order is one less than the order of $y^{(r)}$, and from which $y^{(r)}$ may be obtained by a single differentiation. The existence theorem referred to may be stated as follows:

THEOREM II. *Suppose that in the system of partial differential equations*

$$\frac{\partial y^{(j)}}{\partial u_k} = \sum_{i=0}^{n-1} a_i^{(j,k)} y^{(i)}, \quad (j = 0, 1, \ldots, n-1;\ k = 1, 2, \ldots, p;\ y^{(0)} \equiv y)$$

the derivatives $y, y^{(1)}, y^{(2)}, \ldots, y^{(n-1)}$ *form a normal set. Suppose further that in the closed region A the coefficients* $a_i^{(j,k)}$, *which are functions of the p independent variables,* $u_1, u_2, \ldots, u_p$, *satisfy identically the integrability conditions*

$$\frac{\partial a_\nu^{(j,k)}}{\partial u_l} + \sum_{i=0}^{n-1} a_i^{(j,k)} a_\nu^{(i,l)} = \frac{\partial a_\nu^{(j,l)}}{\partial u_k} + \sum_{i=0}^{n-1} a_i^{(j,l)} a_\nu^{(i,k)},$$
$$(\nu, j = 0, 1, \ldots, n-1;\ k, l = 1, 2, \ldots, p).$$

Let $(u_1^{(0)}, u_2^{(0)}, \ldots, u_p^{(0)})$ *be any point of A, and* $y_0, y_0^{(1)}, \ldots, y_0^{(n-1)}$ *be any set of n constants. Then there exists one and only one function y of the variables* $u_1, u_2, \ldots, u_p$ *which satisfies the system of differential equations, and whose derivatives* $y, y^{(1)}, \ldots, y^{(n-1)}$ *take on respectively the preassigned constant values* $y_0, y_0^{(1)}, \ldots, y_0^{(n-1)}$ *at the point* $(u_1^{(0)}, u_2^{(0)}, \ldots, u_p^{(0)})$.

From this theorem may be inferred at once the existence of a fundamental system of n solutions, $y_1, y_2, \ldots, y_n$, such that any other solution of the system of differential equations has the form

$$y = c_1 y_1 + c_2 y_2 + \ldots + c_n y_n.$$

Moreover, any function of this form is a solution of the completely integrable system; this system is therefore a natural generalization of the ordinary homogeneous linear differential equation of the nth order.

The derivatives $y, y^{(1)}, \ldots, y^{(n-1)}$ which appear in the right-hand members of the differential equations we shall call the *primary derivatives*. We may now define the *Wronskian* of n solutions of the completely integrable system, as the determinant formed from the primary derivatives of these solutions:

$$W \equiv \begin{vmatrix} y_1 & y_2 & \ldots & y_n \\ y_1^{(1)} & y_2^{(1)} & \ldots & y_n^{(1)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ y_1^{(n-1)} & y_2^{(n-1)} & \ldots & y_n^{(n-1)} \end{vmatrix}.$$

The Wronskian just defined has properties similar to those met with in the theory of an ordinary differential equation. Thus, it may be shown without difficulty that

$$\frac{\partial W}{\partial u_k} = W \sum_{j=0}^{n-1} a_j^{(j,k)}, \qquad (k = 1, 2, \ldots, p)$$

so that we may determine by a quadrature a function f such that

$$\frac{\partial f}{\partial u_k} = \sum_{j=0}^{n-1} a_j^{(j,k)}, \qquad (k = 1, 2, \ldots, p)$$

Therefore, *the Wronskian W of n solutions of the completely integrable system may be determined by a quadrature from the coefficients of the system, and is given by the expression*

$$W = \text{const. } e^f.$$

This is a generalization of the theorem of Abel for an ordinary homogeneous linear differential equation of the nth order.

We shall state one more theorem, the analogue of a familiar one concerning an ordinary differential equation. The completely integrable systems to which it applies are of somewhat less generality than those for which the existence theorem has been given.

THEOREM III. *Suppose the completely integrable system considered in Theorem II has in addition the following properties:*

1°. *The set of primary derivatives is such that, if $y^{(i)}$ be any one of the set, then all the derivatives of lower order from which $y^{(i)}$ may be obtained by differentiation also belong to the set.*

2°. *All the first derivatives of the primary derivatives exist for each of the np coefficients $a_j^{(j,k)}$ $(j = 0, 1, \ldots, n-1;\ k = 1, 2, \ldots, p)$.*

Then the system of differential equations may be transformed in but one way into a system

$$\frac{\partial \bar{y}^{(j)}}{\partial u_k} = \sum_{i=0}^{n-1} \bar{a}_i^{(j,k)}\, \bar{y}^{(i)} \qquad (j = 0, 1, \ldots, n-1;\ k = 1, 2, \ldots, p)$$

for which all of the quantities

$$\frac{\partial \bar{f}}{\partial u_k} \equiv \sum_{j=0}^{n-1} \bar{a}_j^{(j,k)}, \qquad (k = 1, 2, \ldots, p)$$

are zero, by the transformation of the dependent variable $y = \lambda y$, where

$$\lambda = \text{const. } e^{f/n}.$$

This last theorem is of interest in the method developed in recent years by Wilczynski for dealing with questions in projective differential geometry. In fact, the coefficients of the transformed system of differential equations are what he has generally called seminvariants of the original system; the theorem affords a means for calculating these seminvariants in a purely mechanical way. In Wilczynski's method, the geometric problem becomes the study of a completely integrable system of the kind we have been considering.

The results outlined above have been developed at length in a memoir which is to appear in the *Transactions of the American Mathematical Society.*

SYSTEMATIC MOTION AMONG STARS OF THE HELIUM TYPE

By Benjamin Boss

DUDLEY OBSERVATORY, ALBANY, N. Y.

Received by the Academy, March 1, 1916

Several investigators using different methods and different material have shown beyond a doubt that the stars evidence a preference for motion in two opposite directions in the sky. This does not mean that all stars move in one of the two directions, but that there is a stronger tendency for stars to move in the favored directions than in any other.

In such investigations the helium, or B-type, stars have presented considerable difficulties, since their motions are small, and since as a class they are situated at a great distance from the sun. It has seemed desirable, therefore, to devise a method whereby the preference of motion among the helium stars might be determined with some degree of confidence.

In the first place the zone in which all the helium stars lie was mapped off into twelve arbitrary divisions. For each division means were taken of the amount of proper motion in the two co-ordinates right ascension and declination, and these mean values were then subtracted from each proper motion. Thus the center of the velocity-figure was obtained. The rectangular co-ordinates were converted into polar co-ordinates and arranged in the order of their position-angles from the north pole. Then for thirty degree groups, 0°–30°, 10°–40°, etc., means were taken of the position-angles, and sums of the amount of proper motion. With the mean position-angles as abscissae and the sums of proper motion as ordinates, the results were plotted and smooth curves drawn to represent them. Figure 1 shows how well the observations can be fitted by mooth curves. It will also be noted that there is more than one maxi-

mum, though in the cases of the curves 5 and 10 only one maximum is real. That is, in the two cases cited, a slightly different grouping of the data would produce great changes in the curves with the exception of one maximum which is real.

Fig. 1

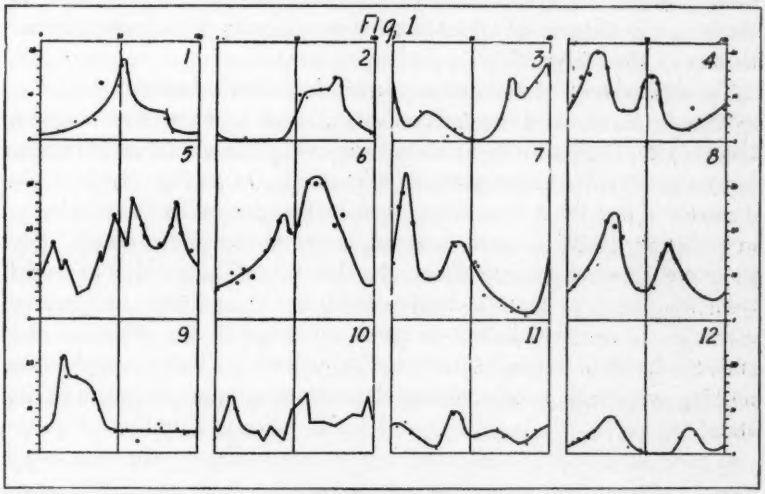


Fig. 2

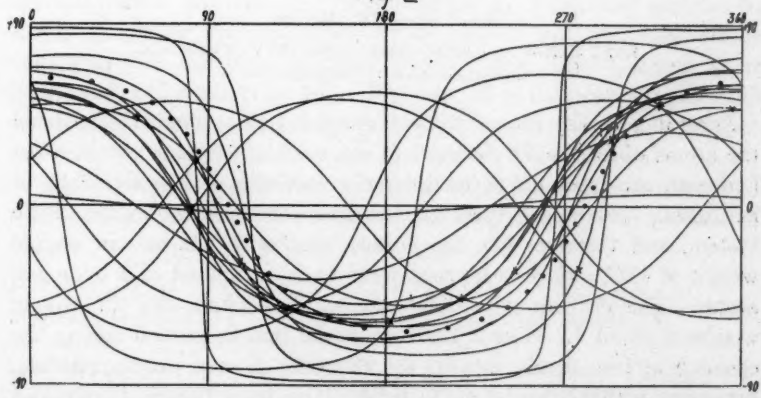


Figure 2 gives a graphic representation of the results. The crosses mark the positions of the centers of the areas treated. From these centers great circles were drawn in the direction obtained by reading off the position-angles of the maxima from the curves of figure 1. The figure then shows the paths along which there is a strong tendency for

stars of the helium type to move. The black dots trace the path of the Milky Way. The paths sharply define a plane which is that of the stars used instead of the plane of the Milky Way, but in addition there are considerable tendencies of motion which carry the helium stars into other regions. In some cases this tendency seems to be the greater of the two. In the care of area 4 there is no tendency for a preference for motion in the Milky Way.

To summarize the conclusions drawn from the investigation of the systematic motions of the helium stars, there appears to be a strong tendency for these stars to move in their own plane, which should therefore be preserved, at least until the next step in the star's evolution. As a matter of fact the A-type stars, supposedly representing the next stage in evolution, exhibit a strong tendency to crowd toward this plane. But there are likewise strong tendencies for the stars of helium type to depart from the plane, so that the tendency for the stars to spread in every direction, so clearly manifest in advanced stages in the evolution of a star, has its birth in the helium stage of evolution. There is apparently nothing systematic in the motions directed away from the plane of the stars.

THE ABUNDANCE OF THE ELEMENTS IN RELATION TO THE HYDROGEN-HELIUM STRUCTURE OF THE ATOMS

By William D. Harkins

KENT CHEMICAL LABORATORY, UNIVERSITY OF CHICAGO

Received by the Academy, February 26, 1916

According to the theory already presented in a number of papers[1] the atoms of all the 91 elements of our ordinary system heavier than hydrogen are built up as intra-atomic (not chemical) compounds of hydrogen. The first of these 91 elements, helium, is the second in the system, and therefore has the atomic number 2. It has an atomic weight of 4.00, and may be considered to be composed of 4 hydrogen atoms. The element of atomic number 3, lithium, has an atomic weight of about 7. Now it has been found that in general among the elements of low atomic weight, the elements of even atomic number, beginning with helium, seem to be built up from helium atoms, and therefore may be said to have the general formula $n\mathrm{He}'$, where the prime is added to indicate that these elements are intra-atomic, not chemical, compounds. The odd numbered elements, beginning with lithium, seem in general to have the formula $n\mathrm{He}'+\mathrm{H}_3'$. Thus the elements seem to fall into two series which may be called the *even* and the *odd* series, or the

helium and the lithium series, if each series is named for its first member. However, it should be noted that while the formula for the helium series is $n\text{He}'$, that for the lithium series is not $n\text{Li}'$.

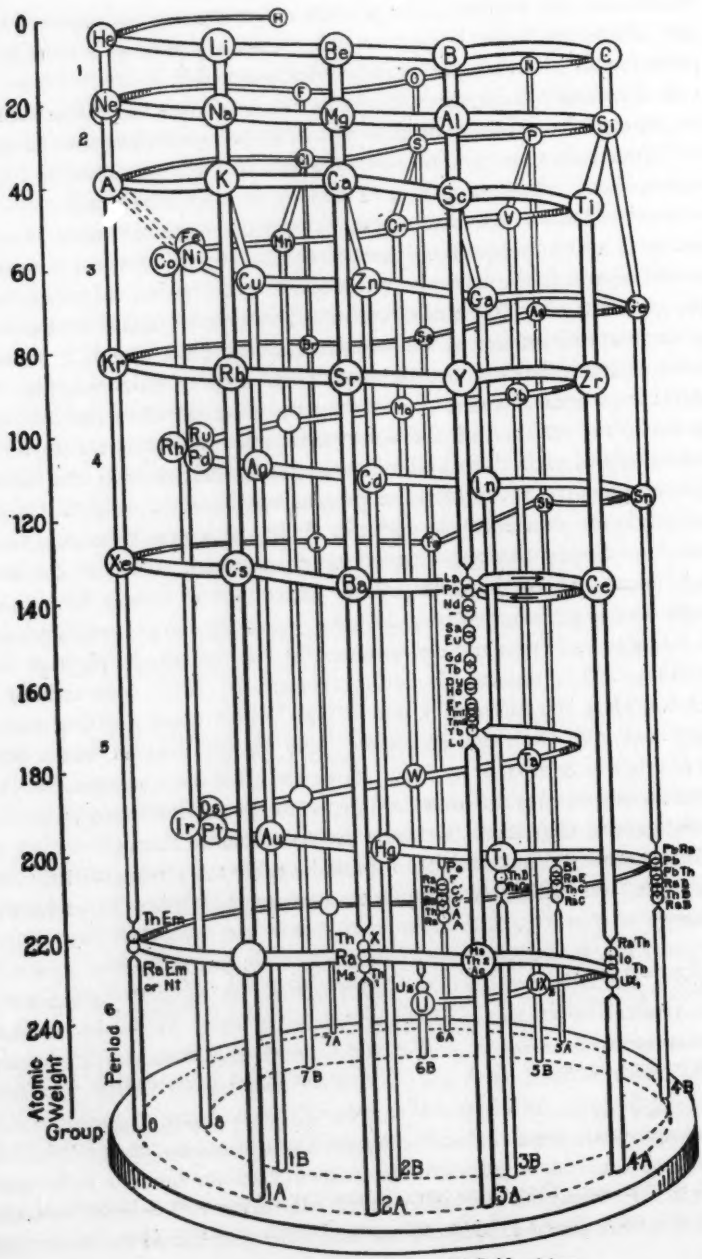
If the theory is correct it might be expected that some characteristic of the elements could be found, with respect to which there is a difference from odd to even and from even to odd, or in other words the elements should show variations in periods of 2 elements each.

In order to have a basis for the comparison of the elements in the study of this problem there has been constructed in space a periodic model of which the accompanying figure is a drawing. In this model the elements are represented by balls strung on a spiral in the order of their atomic numbers, which have recently been found to be much more characteristic of the elements than their atomic weights. The spiral is so arranged that the balls representing the elements belonging to one group and having the same maximum valence are strung on the same vertical rod. The balls are set at such heights that the vertical distance from the top down represents the atomic weight. This is essential, for otherwise the different kinds of atoms of one element, called by Soddy 'isotopes' cannot be represented. Thus in the lower right hand part of the table, on the lower part of Group 4B, the element lead is represented by 6 isotopes, with the atomic weights listed, as follows: lead from radium (uranio-Pb) 206.1; lead, 207.2; lead from thorium, 208.1; radium D, 210.1; thorium B, 212.1, and radium B, 214.1. Thus the different kinds of lead, which seem identical chemically and give the same spectrum, have atomic weights which differ by as much as 8 units, or by 4%. However, all of these isotopes have the same atomic number, 82, or according to the theory developed by various investigators, they have the same positive nuclear charge.

When arranged in this form of periodic table the elements other than hydrogen and helium are found to arrange themselves in periods as follows:

1. First short period	Li – Ne: $8 = 2 \times 2^2$ elements	Cycle $1 = 4^2$
2. Second short period	Na – A : $8 = 2 \times 2^2$ elements	
3. First long period	K – Kr: $18 = 2 \times 3^2$ elements	Cycle $2 = 6^2$
4. Second long period	Rb – Xe: $18 = 2 \times 3^2$ elements	
5. First very long period	Cs – Nt: $32 = 2 \times 4^2$ elements	Cycle $3 = 8^2$
6. Second very long period	Eka – Cs: —— (incomplete)	

It is thus seen that these periods and cycles make up a numerical system of a remarkably simple form, and it seems evident that this system



Periodic Table by W.D. Harkins

must express something inherent in the structure of the atoms. However, what it is desired to emphasize here is that nearly all of the physical properties of the elements vary in periods which are either the same or nearly the same as these. The chemical properties also vary in rather long periods, which in the case of the short periods 1 and 2, are identical with those given.

From this it is seen that both the chemical and the physical properties of the elements vary in periods which are long in comparison with the change in periods of 2 elements as indicated by the division of the elements into the odd and even series. If now neither the physical nor the chemical properties vary according to these extremely short periods, what, it may be asked, is left which can so vary?

Now it might easily be shown that the hydrogen-helium system of the structure of the elements, which divides them into the odd and even series, is in reality more directly applicable to the structure of the nuclei of the atom than to the atom as a whole. If then the Rutherford theory that the nuclei of atoms are extremely minute, is used as a basis for reasoning, it would be expected that the variations in the structure of the nuclei should not cause variations in the properties of the elements except in so far as they influence the nuclear charge. This nuclear charge has been assumed to be equal to the atomic number, and therefore rises with perfect regularity from odd to even or from even to odd. It seems probable that the number of electrons external to the nucleus is equal to the nuclear charge, and that it is the change in their number and arrangement which causes the physical properties to vary according to the periods listed above. This question has been discussed in a previous paper.[2]

It might be expected, however, that the composition of the nucleus should affect its own stability, which from radioactive evidence means the stability of the atom. From this standpoint it might be reasonable to suppose that the atoms of one of the series, the even or the odd, should be more stable than those of the other. Now unfortunately there is no known method of testing the stability of the lighter atoms, but it might seem, at least at first thought, that the more stable atoms should be the more abundantly formed, and to a certain extent this is undoubtedly true. If then, at the stage of evolution represented by the solar system, or by the earth, it is found that the even numbered elements are more abundant than the odd, as seems to be the case, then it might be assumed that the even numbered elements are on the whole the more stable. However, there is at least one other factor than stability which must be considered in this connection. The formula of the even num-

bered elements has been shown to be $n\mathrm{He}'$, which may be written $n(4\mathrm{H}')'$. Now, since the formula for the odd numbered elements is $n\mathrm{He}'+\mathrm{H}'_3$, or $n(4\mathrm{H}')+\mathrm{H}_3'$, it is evident that, if the supply of H_3 needed by the elements was relatively small at the time of their formation, not so much material would go into this system. This would be true whether the H_3 represents three atoms of hydrogen or one atom of some other element. With regard to the latter alternative, it is at least remarkable that the H_3 occurs 11 times in the system for the first 27 elements, while H_2 and H each occur only once, and it may also be mentioned that Fabry and Buisson[3] have by interference methods determined the atomic weight of nebulium to be 2.7, and this they think indicates that its real atomic weight is 3. Also, Campbell[4] has found that in the nebula N. G. C.[4] Index 418, situated in the southern part of the constellation of Orion, the nebulium spectrum is found farther from the interior than that of helium, while the hydrogen spectrum extends out to a much greater distance still. This, he thinks, indicates that the atomic weight of nebulium lies between the values for hydrogen (1) and helium (4).

In studying the relative abundance of the elements the ideal method would be to sample one or more solar systems at the desired stage of evolution, and to make a quantitative analysis for all of the 92 elements of the ordinary system. Since this is impossible, even in case of the earth, it might be considered that sufficiently good data could be obtained from the earth's crust, or the lithosphere. However, it seems probable that the meteorites represent more accurately the average composition of material at the stage of evolution corresponding to the earth than does the very limited part of the earth's material to which we have access. At least it might seem proper to assume that the meteorites would not exhibit any special fondness for the even numbered elements in comparison with the odd, or vice versa, any more than the earth or the sun as a whole, at least not unless there is an important difference between these two systems of elements, which is just what it is desired to prove.

A preliminary study of the most recent analyses of meteorites of different classes showed that, either for any one class or for the meteorites as a whole *the even numbered or helium system elements are very much more abundant than those of the odd numbered or lithium system.* For a more detailed study use was made of the data collected by Farrington,[5] who suggests that the average composition of meteorites may represent the composition of the earth as a whole.

The results obtained by averaging the analyses of 318 iron and 125 stone meteorites, 443 in all, show that the first seven elements in order of abundance are iron, nickel, silicon, magnesium, sulphur, and calcium;

and not only do all of these elements have even atomic numbers, but in addition they make up 98.6% of the material of the meteorites. Of the remaining elements present to a great enough extent to have an appreciable effect upon the percentage values, 7 are odd and 5 are even, but in all only 1.22% are odd numbered, while 98.78% are even.

Of the iron meteorites 99.22% of the material is made up of even numbered elements, and of the stone meteorites, 97.50%. While the results for the earth's lithosphere are not so striking, they still show the same general tendency very strongly; for, of the six most abundant elements, only aluminum is odd numbered, and the elements of even atomic number make up about 86% of the material. The only odd numbered elements other than hydrogen present in the lithosphere in amounts over 0.2% are aluminum, sodium, and potassium.

Table 2 gives the average composition of iron and stone meteorites, arranged according to the periodic system. The numbers before the symbols represent the atomic numbers, and the numbers underneath give the percentage of the element. It will be noted that *the even numbered elements are in every case more abundant than the adjacent odd numbered elements*. The helium group elements form no chemical compounds, and are all gases, so they could probably not remain in large quantities in meteorites. For this reason, and also because the data are not available, the helium or zero group is omitted from the table. The only criticism which could be made of the system of averaging, which is that of including all accurate analyses, is that it places undue emphasis upon the iron as compared with the stone meteorites. However, since the two relations shown in Table 1 are true for each class of meteorites separately it is evident that they will be true whatever system of averaging may be chosen.

TABLE 1

AVERAGE COMPOSITION OF METEORITES ARRANGED ACCORDING TO THE PERIODIC SYSTEM

SERIES	GROUP ODD 1	GROUP 2 EVEN	GROUP 3 ODD	GROUP 4 EVEN	GROUP 5 ODD	GROUP 6 EVEN	GROUP 7 ODD	GROUP 8		
								Even	Odd	Even
2				6C 0.04%		8O 10.10				
3	11Na 0.17%	12Mg 3.80%	13Al 0.39%	14Si 5.20%	15P 0.14%	16S 0.49%				
4	19K 0.04%	20Ca 0.46		22Ti 0.01		24Cr 0.09	25Mn 0.03	26Fe 72.06	27Co 0.44	28Ni 6.50
	29Cu 0.01									

If attention is now turned to the heavier elements as shown in the model, it is seen that the five unknown elements eka-caesium, eka-manganese 1, eka-manganese 2 (dwi-manganese), eka-iodine, and eka-neodymium, have *odd* atomic numbers. (There is some doubt as to the discovery of thulium 2.) Not only are the unknown elements odd numbered, but among the radio-active elements, if the most stable isotope of each element is used for the comparison, the *odd* numbered elements are much less stable than the adjacent elements of even number.

If we consider the rare-earths—the elements which are most similar chemically, while at the same time their atomic numbers change in steps of one—the same result is obtained. In the following table, which includes, besides the rare-earths a number of elements adjacent to them, the letter *c* indicates common in comparison with the other elements in the table, and *r* indicates rare. *cc* represents very common, etc. The comparison is only a rough one, but it is sufficiently accurate for the purpose for it indicates that in every case the even numbered element is more abundant than the adjacent odd numbered element.

TABLE 2

ATOMIC NUMBER	ABUNDANCE	ELEMENT	ATOMIC NUMBER	ABUNDANCE	ELEMENT
55	c	Caesium	63	rr	Europium
56	ccc	Barium	64	r	Gadolimium
57	c	Lanthanum	65	rrr	Terbium
58	cc	Cerium	66	r	Dysprosium
59	r	Praseodymium	67	rrr	Holmium
60	c	Neodymium	68	r	Erbium
61	rrr	Unknown	69	rr	Thulium
62	c	Samarium			

The above results may be summarized by the statement that in the evolution of the elements much more material has gone into the even numbered elements than into those which are odd, either because the odd numbered elements are less stable, or because some constituent essential to their formation was not sufficiently abundant, or both.

It is easy to see too that most of the material has been used up in the formation of the lighter elements. Table 2 shows that in the meteorites the most abundant elements are oxygen in series 2, the elements of series 3 except neon, and the members of the first eighth group triad (iron, cobalt, nickel). Clarke[6] has found that just these same elements are the most abundant in the lithosphere, although in the lithosphere potassium and calcium in series 4 are also moderately abundant. If the lithosphere were considered alone it might be considered that the abun-

dance of these elements is due to changes which have taken place in the lithosphere, or to the rising to the surface of the lighter elements, but these objections are not so valid when the meteorites are found to show the same relations. The density of the earth's surface rock averages between 2.70 and 2.75, the mean density of the earth is 5.516, and the density of its center has been estimated by Lunn[7] as 9.6 on the basis of Roche's law of density, and on the supposition that the chemical composition of the earth is uniform. Stone meteorites vary in density from 2.5 to 5, and iron meteorites from about 6 to more than 8, with an average density of 7.8. According to Lunn[7] the pressure at the center of the earth is 2,800,000 atmospheres, and a possible central temperature is 16,610° when both are calculated on the basis of Roche's law, $\rho = \rho_0 (1 - cx^2)$. It seems probable that this law is much more in accord with the behavior of material than the simple Laplacian form usually used. Some writers have argued from the data that the center of the earth is mostly iron. However the extremely long range of extrapolation above the experimental values in both temperature and pressure, makes it seem impossible to get results in this connection which have the least value, however desirable it would be for such a problem as the one presented here if such a deduction could be properly made. Perhaps, then, the most that can be said is that in the three classes of material, the lithosphere, the stone meteorites, and the iron meteorites, in spite of variations in density from 2.5 to 8, the same two rules are found to hold, that (1) the even numbered elements, and (2) the elements of low atomic number and low atomic weight, are the elements which occur in abundance.

If an artificial line of division is made just after the first eighth group in the periodic model so as to classify the first 29 elements as of low atomic number and atomic weight, and the remaining 63 elements as of high atomic weight, then the following table, based upon data from analyses listed by Farrington and Clarke may be presented to emphasize the importance of the former class.

TABLE 3

PROPORTION IN VARIOUS MATERIALS OF THE ELEMENTS OF LOW ATOMIC NUMBERS

Material	Percentage of Elements with Atomic Numbers 1–29	30–92
Meteorites as a whole	99.99	0.01
Stone Meteorites	99.98	0.02
Iron Meteorites	100.00	0.0
Igneous Rocks	99.85	0.15
Shale	99.95	0.05
Sandstone	99.95	0.05
Lithosphere	99.85	0.15

It may be said that, so far as the abundance of the elements goes, the system seems to play out at the end of the first eighth group in the periodic system. It may be of interest to note here, what has been pointed out in former papers, that it is just at this point in the system that the atomic weights cease any longer to be very close to whole numbers, as they are for the lighter weight elements. Also just at this point the exact formula given for the elements ceases to hold well. These facts do not mean however, that the system fails beyond the iron group of elements; for it is just among the heaviest elements that it received its verification by the actual decomposition of the elements into helium.

The complete paper of which this is an abstract, will be published later. I wish to thank Prof. C. W. Balke of the University of Illinois for suggestions in regard to the relative abundance of the elements of the rare earth group.

[1] Harkins and Wilson, These PROCEEDINGS, 1, 276 (1915); *J. Amer. Chem. Soc.*, 37, 1367–1421 (1915); *Phil. Mag.*, 30, 723 (1915).

[2] Harkins and Hall, *J. Amer. Chem. Soc.*, 38, 186–8, 203–5, 211–14 (1916).

[3] H. Buisson, Ch. Fabry, and H. Bourget, *Astrophys. J.*, 40, 256 (1914). See also Dempster, *Ann. Physik.*, 47, 792 (1915).

[4] Wright, These PROCEEDINGS, 1, 590–5 (1915).

[5] Farrington, Publications 120 and 151, Field Columbian Museum, Chicago.

[6] Clarke, *The Data of Geochemistry*, Bulletin 491, Department of the Interior (1911); see also Bulletin 616 (1915).

[7] Lunn, in 'Tidal and Other Problems,' pp. 201–18, Carnegie Institution (1909).

THE GENETIC RELATIONS OF CERTAIN FORMS IN AMERICAN ABORIGINAL ART

By Clark Wissler

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK CITY

Received by the Academy, March 2, 1915

One of the most difficult problems in anthropology has been the working out of successive steps in the origins of particular traits of culture. The most intensive effort seems to have been made in studies on the evolution of decorative designs. By arranging designs found upon prehistoric or other pottery in order of their increasing conventionality, series have resulted, showing a clearly realistic drawing at one end and an almost entirely geometrical one at the other. Such series suggest that all these forms were arrived at by first drawing from real life and then by successive conventionalizations arriving at a pure geometric form. The weak point in this interpretation is that there are no means of dating the units of the series, their arrangement being merely a matter of selection on the part of the observer. There are still

other obvious objections to the interpretation, so that the tendency of the critical is to reject these conclusions. Somewhat analogous attempts have been made in the study of industrial arts and technology, but with equally unconvincing results. Consequently, as the case stands today, we can point to scarcely a single example in which the life history of a trait can be satisfactorily demonstrated in objective data.

In the course of some technological studies in the American Museum of Natural History, the writer observed that certain structural styles of skin clothing among the Indian tribes of the Mississippi and Great Lakes areas were clearly due to the forms of the original materials. The skins of deer were used and practically always removed from the animal in the same way. Whole skins were then combined to form a garment, their natural outlines being preserved, but usually trimmed to a symmetrical form. That the cut, or style, thus resulting became a recognized feature in the native mind is shown by its survival after cloth was introduced by traders. Here the form of the material did not lend itself to the style but nevertheless was cut to conform to it. Hence, we have a case in which the evidence for the genesis of a trait is satisfactory.

Next our attention was turned to the decorations upon these costumes. Here it can be shown that these same stylistic lines, determined by the contours of the material, were followed in the embroidered decorations, resulting in peculiar curved designs. Thus on the old specimens in our collections, the design follows the cut of the skin material, but upon the modern ones it is repeated upon an even unbroken surface. So without going into details we may state that satisfactory proof can be given to show that this particular design rose from the decoration following the contour of a part of the garment.

The investigation was then extended to moccasin decoration in the same geographical area. In this case satisfactory evidence can be found for the same kind of genetic relation between three different styles of decoration and as many different types of structure. For instance, north of the Great Lakes, for a considerable distance, east and west, the moccasin is made by folding a piece of skin up over the foot and joining on the top and at the heel. The shape of the foot prevents its covering the entire instep, leaving a U-shaped space. This is closed by an insert. The decoration is placed upon this insert, for which there are good technological reasons, and so has in its entirety a U-shape. Then among some of the neighbors of these tribes, particularly the Blackfoot, we find a similar decoration upon a mocassin of an entirely different

pattern and one in which it bears no necessary relation to the structure. Yet these Indians are consistent enough to make a false insert upon which the decoration is placed. In short, there are very strong reasons for concluding that the Blackfoot borrowed the decoration from their northern neighbors and that these tribes arrived at it by adjusting the decoration to the structure.

Another cut of moccasin among some tribes south of the Great Lakes requires no insert on the instep, but has a long unsightly puckered seam, extending down the middle of the foot to the toe. This is usually concealed by overlaying with a long narrow band of embroidered skin. This style extends over into the tribes of the Plains to the west where we find it upon moccasins of a pattern having no seams to hide.

Again, the Apache of New Mexico and Arizona have a moccasin with a long narrow insert reaching down the top of the foot to the toe. This gives two converging seams which are concealed by fringes and very narrow embroidered bands. Then among their northern neighbors we find moccasins without any insert whatever bearing exactly the same decoration.

Thus, in moccasin decoration we find three different examples of decorative designs developed from the structure.

We may summarize this investigation as revealing several good examples of the genesis of specific decorative designs. With one possible exception, they differ from the previous genetic studies of design in that the origin was not strictly in attempts at realistic art but merely grew out of attempts to embellish surfaces of fixed contour and to conceal unsightly lines.

The data in full will appear in the *Anthropological Papers of the American Museum of Natural History.*

THE SITUATION IN REGARD TO ROWLAND'S PRELIMINARY TABLE OF SOLAR SPECTRUM WAVE-LENGTHS

By Charles E. St. John

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Received by the Academy, March 6, 1916

The appearance of the *Preliminary Table of Solar Wave-lengths* marked an epoch in the history of spectroscopy and for twenty years it has been an instrument of precision in the hands of the solar and stellar investigator, to such a degree that it has become an integral part of the literature of spectroscopy. In the transition to the international system, all that is of permanent value in this work of great magnitude and im-

portance should be kept and it becomes necessary to discriminate between that which is possible to retain and that which must pass.

Several attempts have been made to determine in some general way the systematic differences between the Rowland wave-lengths and later determinations based upon the international system. Fabry and Perot[1] plotted the ratio $(\lambda R)/(\lambda \text{ international})$. To reduce the Rowland wave-lengths to the new standard, they proposed to divide the Rowland values by the corresponding ratios read from the curve. Hartmann[2] suggested the reverse procedure, namely, that wave-lengths expressed in the new system be multiplied by these ratios, reducing them to the approximate Rowland values and thus minimizing the break with the historical standard. Instead of the ratios suggested by Fabry and Perot, Albrecht in a recent paper[3] uses as ordinates the quantities, Rowland minus international, and from the mean curve derives directly the systematic difference for any line.

In many present-day discussions of solar problems, the third decimal place, when wave-lengths are expressed in angstroms, is drawn upon for the requisite data. One purpose of the present communication is to direct attention to some observations which show that a general transformation from one system to the other is a matter of the greatest difficulty if the required degree of precision is to be reached, even though the relative wave-lengths in each system were free from error, and that statistical comparison between different systems is a procedure fraught with the possibilities of introducing residuals that may be quite misleading.

For several years direct comparison between the lines of the iron arc and the solar spectrum have been in progress at this Observatory, and recently more precise determinations of the wave-lengths of iron lines dependent upon arc conditions have been made. The sun-arc comparisons show that for the lines of the same pressure group the displacements between sun and arc are, among other relations, a function of the line intensity.[4] The differences, then, between the Rowland and the international wave-lengths must be unequal even for lines in the same group and spectral region. In Table I are shown the data for two strong lines situated between weaker lines of the same group. The larger sun-arc displacements paralleling the larger Rowland-International differences for the strong lines are typical and the sun-arc data accord with the mean from a long series of determinations.

Similar systematic differences appear when lines of different groups, but in the same region, are considered, as shown in Tables II and III.[5]

TABLE I

Iron Lines of the Same Group and Spectral Region but Differing in Solar Intensity

Group	λ Rowland	Intensity	λ International	R–I	Sun–Arc
b	4337.216	5	.052	0.164	+0.004
b	4352.908	4	.740	0.168	+0.005
b	4383.720	15	.548	0.172	+0.012
b	4404.927	10	.753	0.174	+0.010
b	4427.482	5	.314	0.168	+0.003
b	4443.365	3	.198	0.167	+0.003
Mea		12		0.173	+0.011
Mea		4		0.167	+0.004
Intenity 12–Intensity 4				+0.006	+0.007

TABLE II

Iron Lines of Group a and d in the Same Region

Group	λ Rowland	International	R–I	Sun–Arc
d	5340.121	.945	0.176	−0.003
a	5341.213	.029	0.184	+0.009
d	5393.375	.178	0.197	−0.006
a	5397.344	.135	0.209	+0.008
Mean Group a			0.196	+0.008
Mean Group d			0.186	0.004
Mean Group a–Group d			+0.010	+0.012

TABLE III

Iron Lines of Groups a and e in Same Region

Group	λ Rowland	International	R–I	Sun–Arc
e	5407.357	.136	0.221	+0.015
a	5405.989	.780	0.209	+0.006
e	5411.124	.904	0.220	+0.016
e	5415.416	.195	0.221	+0.026
e	5424.290	.064	0.226	+0.026
a	5429.911	.702	0.209	+0.007
a	5434.740	.529	0.211	+0.009
Mean Group a			0.210	+0.007
Mean Group e			0.222	+0.021
Mean Group e–Group a			+0.012	+0.014

It is evident from the data given that no factor of transformation nor any curve can yield true differences between the Rowland and international wave-lengths for all lines even for a limited spectral region.

A comparison has recently been made between the separations of close pairs as given in the Rowland tables and as measured upon plates of higher dispersion than that used by Rowland. For pairs consisting of lines of intensities 3 and 4, whose separations are 0.0 to 0.1 A, 0.1 to 0.2 A, and 0.2 to 0.35 A, the Rowland values exceed those found at

Mount Wilson by 0.011 A, 0.007 A, and 0.004 A, respectively. These systematic differences furnish new ground for not adopting any form of operator for transforming the wave-lengths in Rowland's Preliminary Table to the international standard, and indicate some possible pitfalls incident to statistical methods of comparison.

Though the Rowland wave-lengths cannot be transformed so as to reproduce rigorously the wave-lengths expressed in the international system, it does not follow that the *Preliminary Table of Solar Wave-lengths* has outlived its usefulness. It will be some years before complete tables of solar wave-lengths, based upon the new standards, will be available, and even then the Rowland table will serve indefinitely as a reference for other things than wave-lengths. It has been assumed that the accidental errors in it are considerably greater than ± 0.01 A, but recent comparisons between certain lines in Rowland's table and their wave-lengths as measured here upon plates of a very high dispersion, 1 mm. = 0.3 angstrom, using as standards neighboring free-standing lines, show that this is an over-estimate. For 54 lines in pairs with separations between 0.25 and 0.50 angstrom, the mean variation from Rowland's values is ± 0.003 angstrom. As more than 200 lines were used as standards, it appears that, for the types of lines involved, the accidental errors in the Rowland wave-lengths are much less than 0.01 angstrom. It is probable, as Frost and Adams remark,[6] that errors of this magnitude occur but rarely and mainly then for lines whose measurement is inherently difficult, such as the very weak and the strong shaded lines. When it was first noticed that the differences between the Rowland wave-lengths in the Preliminary Table and those found by the newer arc determinations were not constant over even a short spectral region, it appears to have been assumed that the large variations were due to errors in the Rowland values, but data similar to those reported in Tables I, II, and III show that the apparent discrepancies represent real differences in the behavior of lines and tend to increase rather than destroy confidence in the accuracy of the relative wave-lengths of lines not presenting special difficulties of measurement and sufficiently separated from others to be within the power of spectrographs in common use.

[1] Ch. Fabry and A. Perot, *Astroph. J.*, **15**, 272 (1902).

[2] J. Hartmann, *Astroph. J.*, **18**, 167 (1908).

[3] Sebastian Albrecht, *Astroph. J.*, **41**, 333 (1915).

[4] Charles E. St. John, *Mt. Wilson Contr.*, No. 93, p. 35; *Astroph. J.*, **41**, 63 (1915).

[5] Charles E. St. John and L. W. Ware, *Mt. Wilson Contr.*, No. 61, pp. 31–32; *Astroph. J.*, **36**, 45–46 (1912).

[6] E. B. Frost and W. S. Adams, *Publications of Yerkes Observatory*, vol. 2, p. 155.

CHANGES IN THE FORM OF THE NEBULA N. G. C. 2261

By Edwin P. Hubble

YERKES OBSERVATORY, UNIVERSITY OF CHICAGO

Received by the Academy, March 9, 1916

A comparison of photographs has established changes in structural detail in the nebula N.G.C. 2261 (h 399) R.A. 6^h 35^m, Dec. $+8°50'$. This nebula is the finest example of a cometary nebula in the northern skies. The nucleus is known as the variable star R Monocerotis, said to range from magnitudes 9 to 13.5, with an irregular period. Lassell states that the nucleus is not a star, and Professor Barnard confirms this opinion from observations with the forty-inch refractor.

Plates obtained by me with the twenty-four inch reflector during the last six months were compared in the Blink-Mikroskop with an excellent plate taken with the same telescope in March, 1908, by Mr. F. C. Jordan. In the interval, the following edge of the nebula has bulged out to a greater convexity; a bright portion of the nebulosity just north of the nucleus has shifted about 5″ toward the east; the north preceding part has moved toward the south following. In the center, however, is a sharply defined brighter wedge-shaped portion pointing to the east, which shows no motion.

In compliance with the request of the Director of this Observatory, Mme. Dorothea Roberts had the great kindness to prepare and send us both positive and negative copies of the plate of this nebula taken by the late Dr. Isaac Roberts at Starfield on January 27, 1900, and shown in *Knowledge*, vol. 24, p. 181. These amply confirm the reality of the phenomena and further establish that the motion is progressive both in direction and amount. The first impression is that the nebula is turning about its own axis after the manner of a top, and there is some indication of a helical motion toward the nucleus. The observed shifts seem to be rather of mass than of illumination and are independent of the variability of the nucleus.

Such changes are so novel that a question at once arose as to whether they might not be due to differences in exposure times or other photographic conditions rather than in the nebula itself. Fifteen plates obtained during the last six months under widely varying conditions of steadiness, transparency, and exposure time, agree perfectly in detail when compared among themselves. Of these, three were taken on the same night, with half, full, and double the normal exposure time, and show no differences other than the symmetrical building up of the image. Further, Director Frost and Professor Barnard have examined the plates

and give me authority to say that in their opinion the changes are in the nebula itself.

The position of the nebula is in a dark lane in the sky, connected with the nebulosity around 15 Monocerotis. This seems significant in that the two nebulae in which variability has certainly been established both lie in dark regions. These are N.G.C. 1555, known as Hind's variable nebula (close to T Tauri), and N.G.C. 6729 in Corona Australis. In one other case, that of the planetary N.G.C. 7662, Professor Barnard has found that the nucleus varies through several magnitudes.

Preceding the nucleus (R Monocerotis) by 4.4 and north 97″ is a star somewhat fainter than the 15th magnitude with a proper motion of the order of 30″ per century. North following 9′ and 10′, respectively, are two variable stars whose maxima are at about 15.5 mag.; and north preceding 17′ is still another variable with a range of at least from the 11th to the 17th magnitude; all of which add to the interest of this remarkable region of the sky.

The plates are being measured and a more detailed investigation of the data, with reproductions of the photographs, will be published in the *Astrophysical Journal*.

ON THE EFFECT OF REMOVAL OF THE PRONEPHROS OF THE AMPHIBIAN EMBRYO

By Ruth B. Howland

SWEET BRIAR COLLEGE, SWEET BRIAR, VA.

Received by the Academy, March 9, 1916

The following note embodies the results of a series of experiments, performed at Yale University at the suggestion of Dr. R. G. Harrison, on embryos of the spotted salamander, Amblystoma punctatum. The particular problems in mind were first, to determine whether the head kidney or pronephros is a functioning organ necessary to the life of the embryo, and second, to investigate the correlation of the development of this organ with that of other components of the excretory system.

The embryos used for the experiments were nearly of a uniform age, varying slightly in size and degree of development from the condition in which the first loop of the pronephric tubules appears as a slight, ventrally directed curve of the duct, to the stage in which the two funnels, together with the first loop, appear as a broadened Y. (Fig 1.) The tail-bud was clearly defined, and the pronephric swelling distinctly visible. In all cases, however, embryos were used before contraction of the body

muscles began, for contraction not only hindered the operation, but often tore open the wound after successful removal of the kidney.

Two methods were employed in removing the pronephroi. In the first, three straight cuts were made, one beneath and one along each side of the pronephric prominence. The flat of skin thus defined was lifted up and the organ removed from below. In the second and more satisfactory method, a single incision was made dorsal to or immediately over the thickening, the needle inserted, and the tubule raised upward from the ventral side and excised.

Conditions ensuing upon the removal of the head kidney of both sides in Amblystoma larvae show clearly that these organs are necessary to the life of the embryo, although the presence of one pronephros suffices to keep the organism alive and in a healthy condition. All embryos from which both head kidneys had been excised died within eight to twelve days, evidencing during that interval symptoms of weakened heart-action—oedema and effusion into the pericardial and abdominal cavities—presumably brought about by uremic poisoning. Pricking the body wall to relieve the dropsical condition was resorted to in many cases from five to seven days after operating, but it proved ineffective.

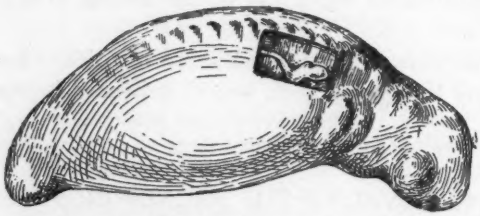


FIG. 1.—AMBLYSTOMA EMBRYO IN THE OPERATING STAGE, SHOWING THE PRONEPHROS EXPOSED AFTER REMOVAL OF ECTODERM. × 13.

The pronephros remaining after the removal of one head kidney evidently takes over the function of excretion usually performed by the two organs, and, concomitant with the increased physiological activity, presents marked morphological changes. The size of the organ which functions alone is greatly increased, indicating the occurrence of compensatory hypertrophy. In the normal pronephros, the walls of the tubules are thick, and consist of cuboidal cells, the central ends of which often bulge out into the narrow lumen. The hypertrophied tubules are thin walled, the cells flattened as is the case in normal larvae of greater age, and the lumen accordingly is nearly twice that of the unoperated specimen. To determine the nature and extent of the change brought about in the functioning pronephros through the removal of the organ on one side, wax models of this organ (× 200) of an operated specimen and of a normal embryo were constructed. The control was chosen from a large number of normal embryos which, on sectioning, showed

the several organs (retina, lens of eye, digestive tract, etc.) to be in a stage of development identical with that in the operated individual. On comparison of the models, the evidence of hypertrophy shown in microscopic examination was strengthened. The model of the operated individual not only showed a considerable increase in length of the coiled tubule as contrasted with the control, but by its size indicated a great increase in volume. The length of the tubules making up the models was determined by taking the average of five measurements. The larger model measures 188 cm. (× 200), the smaller model 155 cm. (× 200), showing an increase of 21 per cent over the normal conditions. Difference in weight also indicates hypertrophy, the normal model weighing 62.9 gr. as contrasted with the larger, which has a weight of 115.9 gr.

The formation of the pronephric glomeruli under normal conditions as outgrowths from the aorta toward the newly forming kidneys suggested the possibility that in the operated specimens the glomeruli of one side might be lacking. The removal of one pronephros has, however, no effect on this development, for in all of the operated embryos both glomeruli were present. The glomerulus on the operated side is not as uniform in size and shape as the normally functioning one, its form evidently being largely determined by the width of the coelomic cavity in this region. The outer layer of the glomerular capillaries and the epithelial lining of the body wall often coalesce. Absence of the pronephros enlarges the cavity on that side, and the glomerulus is then found bridging this cavity as a long strand instead of appearing as a more compact tuft of capillaries.

In the removal of the embryonic pronephros, as much of the tissue was scraped out as seemed possible without disturbing the Anlage of the aorta. In spite of this fact, a large number of the operated embryos possessed well developed anterior and posterior nephrostomes and portions of the funnels, indicating that these structures are probably regenerated from the adjacent coelomic epithelium. Of the sixteen embryos examined, twelve showed well formed anterior funnels and nine had posterior funnels. In one instance the anterior funnel was of a peculiar double form, suggesting the normal condition in those amphibia with three pronephric openings.

The effect of excision of one pronephros on the formation of the segmental duct on the operated side varied widely. The process of development of the non-functioning ducts is carried on only to a limited extent. In the embryos examined, every gradation of development was found from a condition in which the lumen, though small and flattened dorso-

ventrally, appears throughout the entire length, to a condition where only the occasional presence of a few degenerating cells indicates the location of the atrophied duct.

Increased activity of a single kidney also has a definite effect on the segmental duct of that side. Cross sections of the duct of an individual with unilateral operation, when compared with either of the ducts of a normal larva of the same stage, show a marked increase in diameter.

The mesonephroi of both sides develop normally, at least in the early stages, even after excision of one pronephros. Non-development of the segmental duct in one instance left the mesonephric tubules disconnected on that side, and with no outlet for excretory products. No specimens have yet been kept a sufficient length of time to determine the ultimate outcome of this abnormal condition.

In brief then, the following conclusions may be drawn.

1. Removal of both pronephroi in Amblystoma larvae induces conditions leading to oedema and subsequent death, though the presence of one head kidney is sufficient to keep the embryo in a condition of health.

2. Excision of one head kidney brings about an increase in size in the remaining organ, and also in the diameter of the segmental duct on that side.

3. Removal of one pronephros has no essential effect on the development of the pronephric glomerulus of that side, but the segmental duct appears in varying stages of atrophy.

4. Anterior and posterior nephrostomes may regenerate from the coelomic epithelium.

5. Early developmental stages of the mesonephros are normal, even after excision of one head kidney.

ON THE PRESENCE OF A MEDIAN EYE IN TRILOBITES

By Rudolph Ruedemann

NEW YORK STATE MUSEUM, ALBANY

Received by the Academy, March 7, 1916

The entirely extinct sub-class Trilobita of the Crustaceans, comprising about 1840 species divided into some 185 genera, has always held a central place in the phylogenetic history of all classes of Crustacea. It is becoming *not only* more probable that the Ostracoda and Cirripedia were developed from the trilobites, but that also the insect subphylum has taken its origin from this ancient class of arthropods. Some authors derive the Xiphosura and Eurypterida and through them the scorpions and all later arachnidians from the trilobites, and others again, as Patten,

trace the vertebrates back to the Xiphosura and Eurypterida. The phylogenetic importance of the trilobites, which were the dominant animal class in the Cambrian era, is therefore assured, and it is for this reason that observations of new structures in the trilobites are of greater interest than they might otherwise be.

An organ not heretofore recognized in the trilobites is the median or parietal eye on the glabella, and yet the question of its presence or absence in the trilobites is of considerable phylogenetic importance. Beecher, through whose observations we have learned so much of the ventral anatomy of the trilobites, would ally them closely with the Phyllopods (Branchiopoda) and Walcott, who has made us acquainted with an unexpected wealth of Cambrian trilobites and other crustaceans, would directly derive them from primitive Branchiopods correlated for convenience with Apus-like forms. On the other hand we find authors such as Kingsley, Bernard and Jaekel, who advance arguments to prove that the trilobites can be traced back directly to arthropods more primitive than any crustaceans, as for example, the annelids. Bernard would even not place them in the class Crustacea.

In these discussions the absence of median or parietal eyes in the trilobites has been emphasized as a feature distinguishing them from most primitive crustaceans, notably the Phyllopoda, as well as from the Merostomes. It is therefore of interest that the presence of such median eyes can be demonstrated in this group.

The median eye appears, in the majority of cases, as a single tubercle upon the glabella. This tubercle has so far been recognized in upward of thirty genera, among them all genera of the Asaphidae and representatives of most other families of the trilobites. In studying the structure of the tubercle it was found that the median eye presents all stages of development seen in other crustaceans, from mere transparent thinner spots of the test to a lenticular body covered by a thin cornea. The lenticular body is frequently recognized by the pit in the interior cast of the tubercle; it was seen in sections of *Cryptolithus* (*Trinucleus*) *tesselatus* from the Trenton limestone of New York. Here it appears as a glossy, pearly body in the interior cast of the eye tubercle when the thin cornea is removed. In sections it is seen to be composed of the same substance as the matrix, but bounded on the under side by a carbonaceous layer; and it is also observed that the test above the lenticular body is thinned to one-half its normal thickness, thus forming a thin cornea. From the absence of a separate crystalline structure in the lens, and the presence of the carbonaceous division line, we infer that there was no hard chitinous lens that would become a separate center

of crystallization in fossilization, as in the lateral eyes of the trilobites, but one corresponding to that of the parietal eye of other crustaceans, and especially of the phyllopods, which is a lens- or pear-shaped sac, usually filled with water. The parietal eye of the trilobites is hence not at all comparable to the larval ocelli of insects or the parietal eye of *Limulus*, the eurypterids and arachnids in general, where the chitinous integument thickens into an exterior lens, but it agrees well in its structure with the median eye of the crustaceans; the thin black layer at the base of the lenticular body being derived from the pigment of the retina. Whether the lenticular cavity was filled with sea-water or a body fluid is not known, but there are indications that some trilobites may have possessed a pore on the apex of the tubercle, giving the sea-water access to the interior.

Indirect evidence for the visual function of the tubercle is seen in the following facts:

The eye tubercle is the sole prominence on the otherwise smooth glabella in the *Asaphidae*, *Trinucleidae*, etc. It must therefore have a function requiring prominence for its performance. It is always situated at the highest spot of the carapace, either on the apex of the bulging frontal lobe of the glabella as in *Cryptolithus* (*Trinucleus*), or where the glabella abruptly bends downward on the prominent posterior portion between the last glabellar lobes (*Isotelus* and *Asaphus*). It is, further, generally situated between the posterior portions of the lateral or compound eyes. This position is explained by the fact that the median or parietal eye according to its origin is always nearest to the brain and this is, in the phyllopods and most other crustaceans, situated in the dorsal region of the head, beneath or between the lateral eyes. As in the eurypterids, so also in the trilobites, the median eye is often found at the posterior extermity of a distinct crest, extending backward a short distance upon the glabella. This crest is probably an analogue of the 'eye-line' of the lateral eyes of the primitive trilobites, and at least in part, marks the path of the nerve leading to the median eye. As in the crustaceans and eurypterids, the median eye tubercle is relatively largest and most prominent in the earliest growth stages, and in the later stages it may entirely disappear, as in *Isotelus gigas*. It is likewise better developed in the more primitive orders of the trilobites; and the phylogenetically late families of the highest order, the Proparia, viz., the *Calymenidae* and *Phacopidae*, seem to have practically lost the median eye. The Ordovician and Silurian trilobites show well developed median eye tubercles; the Devonian forms lack them; the median eye, as in the higher crustaceans, having been either reduced again to

a transparent spot in the test and wandered inward or become altogether effaced. In the Cambrian trilobites, the tubercle, for the most part, is also absent; the median eye from the present evidence, being still in its most primitive form of one or two transparent spots of the test.

It is a distinct fact pointing to a visual function of the median tubercle that the genera usually considered as blind because of reduced or absent lateral eyes, are apt to show these median eye tubercles most distinctly, as notably *Cryptolithus*, *Trinucleus*, *Dionide*, *Dindymene*, and also *Agnostus*, *Microdiscus* and *Ampyx*, while on the other hand the genera, *Phacops* and *Dalmanites* with their highly developed lateral eyes, show the least trace of the median eye. Its constant presence in a great number of genera is further evidence of its important function; and finally the fact that all lower crustaceans typically possess the median or parietal eye and that for that reason zoologists of standing have already simply assumed the presence of this organ in the trilobites, makes it a reasonable inference that these primitive early crustaceans should have also possessed the median eye, in at least some stages of their evolution, and that is what the writer hopes to have demonstrated.

A fuller account of this investigation is being printed in a New York State Museum Bulletin.

THE NATURE OF MECHANICAL STIMULATION

W. J. V. Osterhout

LABORATORY OF PLANT PHYSIOLOGY, HARVARD UNIVERSITY

Received by the Academy, March 13, 1916

The effects of certain kinds of stimuli can be referred directly to chemical changes which they produce in the protoplasm, but there are other kinds which appear to operate by physical means only. In the latter category are such stimuli as contact, mechanical shock and gravitation. While their action appears at first sight to be purely mechanical, they are able to produce effects so much like those of chemical stimuli that it appears probable that in every case their action must involve chemical changes.

The chief difficulty which confronts a theory of mechanical stimulation appears to be this, How can purely physical alterations in the protoplasm give rise to chemical changes? It would seem that a satisfactory solution of this problem might serve to bring all kinds of stimulation under a common point of view, by showing that a stimulus acts in every case by the production of chemical reactions.

An answer to this question is suggested by some observations of the writer. These were originally made on the cells of the marine alga *Griffithsia Bornetiana*. A cell of this alga is shown in figure 1. Within the cell wall (*a*) is a thin layer of protoplasm (*b*) which encloses the large central vacuole (*e*). The protoplasmic layer includes numerous chromatophores (*c*). The latter contain chlorophyll and a red pigment (phycoerythrin) which is soluble in water. Under normal conditions the surface of the chromatophore is impermeable to the red pigment, which is thus confined to the chromatophore and prevented from escaping into the surrounding protoplasm or into the vacuole.

The writer has observed when one of the larger cells is placed under the microscope (without a cover glass) and touched near one end (with a needle or a glass rod or a splinter of wood) a change occurs in the chromatophores directly beneath the spot which is touched. The surfaces of the chromatophores in this region become permeable to the red pigment, which begins to diffuse out into the surrounding protoplasm. This change begins soon after the cell is touched. As the red pigment diffuses through the protoplasm it soon reaches neighboring chromatophores and it may then be seen that their surfaces also become permeable and their pigment begins to diffuse out. In this way a wave—which may be compared to a wave of stimulation—progresses along the cell until the opposite end is reached.

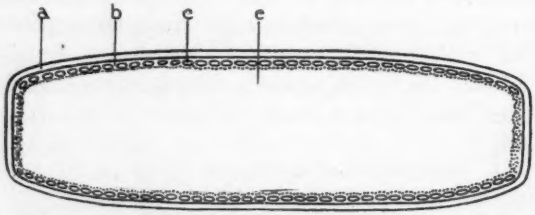


FIG. 1.—A CELL OF *GRIFFITHSIA BORNETIANA* (IN OPTICAL SECTION). *a*, CELL WALL; *b*, PROTOPLASM; *c*, CHROMATOPHORE CONTAINING CHLOROPHYLL AND A RED PIGMENT (PHYCOERYTHRIN) WHICH IS SOLUBLE IN WATER; *e*, VACUOLE FILLED WITH CELL SAP. (DIAGRAMMATIC.)

The rate of propagation of this wave corresponds to that of the diffusion of the pigment. It would seem that at the point where the cell is touched, pigment, and probably other substances, are set free, diffuse out and set up secondary changes as they progress. These changes are doubtless chemical in nature.

The important question then arises, How does the contact initiate the outward diffusion of the pigment or other substances?

It seems to the writer that this may be due to a mechanical rupture of the surface layer of the chromatophore which is either not repaired at all or only very slowly. Many cases are now known in which the

surface layers of protoplasmic structures behave in this way.* If therefore, such structures exist within the cell, it is evident that any deformation of the protoplasm which is sufficient to rupture their surface layers will permit their contents to diffuse out into the surrounding protoplasm. A great variety of cellular structures (plastids, vacuoles, 'microsomes,' inclusions, etc.), possess surface layers of great delicacy and it is easy to see how some of these may be ruptured by even the slightest mechanical disturbance.

It is therefore evident that deformation of the protoplasm may rupture the surface layers of certain protoplasmic structures and cause their contents to diffuse out. If the substances which thus diffuse out meet other substances from which they were separated by the semipermeable surface layer before it was ruptured it is easy to see how reactions may be set up which in certain cells may bring about the responses characteristic of mechanical stimulation. The occurrence of such reactions seems probable, since many cases are known where substances in close juxtaposition are prevented from reacting by the presence of such semipermeable layers; but when these layers are destroyed (by crushing the cells) the reaction at once takes place.

If these processes occur it is evident that purely physical alterations in the protoplasm can give rise to chemical changes. Responses to contact and mechanical stimuli may thus be explained; and since gravitational stimuli involve deformation of the protoplasm we may extend this conception to geotropism.

In this conception of mechanical stimulation the essential things are (1) substances which are more or less completely prevented from reacting by semipermeable surfaces, (2) a deformation of the protoplasm sufficient to produce in some of these surfaces a rupture which is not at once repaired, (3) a resulting reaction which produces the characteristic response to the stimulus.

* In many cases rupture of the plasma membrane causes the protoplasm to disintegrate and mix with the surrounding medium. In other cases the surface layer is at once reconstituted.

HEREDITARY REACTION-SYSTEM RELATIONS—AN EXTENSION OF MENDELIAN CONCEPTS

By R. E. Clausen and T. H. Goodspeed

DIVISION OF GENETICS, COLLEGE OF AGRICULTURE AND DEPARTMENT OF BOTANY, UNIVERSITY OF CALIFORNIA

Received by the Academy, March 11, 1916

The most important as well as the most consistent and intelligible series of Mendelian conceptions are those which Morgan and his associates have formulated on the basis of their extensive studies of heredity in the common fruit fly, *Drosophila ampelophila*. During the progress of their investigations they have observed the origin of over a hundred factor-mutations in this species, and they have determined the hereditary interrelations of a large number of them. They have established, for the fruit fly, the validity of the fundamental conception of Mendelism that the units contributed by two parents separate in the germ cells of the offspring without having had any effect on one another, that long and intimate association in the same chromosomal mechanism does not modify the fundamental constitution or relations in the hereditary mechanism of the units of which it is made up. They have also demonstrated that the known behavior of the chromosomes furnishes a most satisfactory basis for an explanation of the distribution of hereditary units to the germ cells. Furthermore, from the linkage-relations displayed among the factors, Morgan has succeeded not only in demonstrating that the number of groups of factors corresponds to the number of pairs of chromosomes, but he has also succeeded in preparing a map of the relative linear positions of the factors within the chromosomes. It, therefore, follows that, so far as heredity is concerned, the chromosomes are made up of a linear series of loci which bear at least some specific relation to one another as is indicated by this aggregation into chromosomes. Hereditary modifications of characters in the individual depend upon changes in the loci, a particular type of change in some particular locus corresponding to each different character-modification. Now, since a changed locus maintains the same formal relations with the other loci in the system as it does in its normal unchanged condition, it is clear that the chromosome conception of heredity furnishes a consistent explanation of the fundamental nature of allelomorphism and of the mechanistic basis of Mendelian segregation. Further the evidence of somatogenesis seems to indicate that the hereditary units form a physico-chemical reaction-system of which the elements, the loci of the hereditary system, bear more or less specific relations to one

another. In *Drosophila*, for example, the development of the normal abdomen under certain environmental conditions in spite of the presence of the factor for abnormal abdomen in the reaction system indicates the existence of compensatory relations among the factors of the system. Such compensatory relations are even more strikingly evidenced in the case of maize seedlings of the yellow-green chlorophyll reduction type. Normally these die, but under favorable conditions the system is able to overcome the disturbance incident upon the presence of the chlorophyll reduction factor and to go on and develop the normal chlorophyll coloration in the plant. Similarly, the lethal effect of changes in certain loci, the similarity in effect of different changes in the same locus displayed in multiple allelomorphism, the apparently universal significance of the multiple-factor conception of character development, and a variety of other considerations indicate that important physiological relations exist among the loci of the system, and that character expressions depend upon the reaction-system relations of Mendelian factors. The product of somatogenesis, the individual, represents the reaction end-product of such a physico-chemical system working under particular conditions; the specific hereditary differences between individuals of the same species indicate particular differences in some one or more elements in such a reaction-system. Normal Mendelian behavior, then, would follow as a result of hybridization phenomena involving a contrast between a relatively few particular differences within a reaction system which is fundamentally identical in the races under consideration. If in contrast to this type of behavior it should be possible to secure contrasts of fundamentally different reaction systems, then conceivably the elements, although playing definite parts in their own systems, might fail to establish the harmonious inter-relations which are necessary for normal development and reproduction. Such incompatibility of elements would give rise to a peculiar type of behavior in inheritance which could not well be accounted for by the customary formal treatment based on the Mendelian viewpoint. The experimental data which we have collected seem to indicate that such a situation actually does obtain in certain cases of hybridization between distinct species.

For ten years a number of species and varieties of *Nicotiana* have been grown in the University of California Botanical Garden. Among many others this collection has included *N. sylvestris* and a considerable array of varieties of *N. Tabacum*. The varieties of *Tabacum* display notable morphological differences throughout—differences so marked that to regard them as distinct species would be entirely justifiable,

even though they do show evident group relationships. On the other hand *sylvestris* apparently is monotypic and is distinctly different from the *Tabacum* group. Now, the study of a large number of varietal crosses within the *Tabacum* group has demonstrated that most characters are expressed in intermediate degrees in the F_1 hybrids and subsequent segregation in further generations indicates that these phenomena, although complex, are in accord with normal Mendelian expectation. The differences within the *Tabacum* group, therefore, apparently depend upon certain factor differences within a common reaction system. When, however, any one of this array of *Tabacum* varieties is crossed with *sylvestris*, the F_1 hybrid very nearly or completely reproduces on a larger scale the characters of the particular *Tabacum* variety concerned in the hybrid. This has been found true for a number of *Tabacum* varieties; viz. *angustifolia*, *calycina*, *macrophylla*, *macrophylla purpurea*, 'Cavala,' 'Cuba,' and 'Maryland;' descriptions and plates of which based on material grown in the University of California Botanical Garden have been given elsewhere by Setchell. The completeness of the domination of the *Tabacum* parent in the somatogenesis of these F_1 *Tabacum-sylvestris* hybrids is shown particularly in the crosses involving characters which are normally recessive in *Tabacum* variety hybrids. When *calycina*, which produces abnormal, split, 'hose-in-hose' flowers, is crossed with *Tabacum* varieties producing normal flowers, the F_1 hybrids produce the normal type of flowers with few exceptions. On the other hand in marked contrast to the type of behavior in varietal crosses, *calycina* when crossed with *sylvestris* gives an F_1 hybrid which produces only calycine flowers. Similarly, the partially parthenocarpic tendency of 'Cuba,' which is manifested in the retention and normal development of many fruits without pollination, although recessive in varietal crosses, is so impressed on the 'Cuba'-*sylvestris* hybrid that all the fruits mature normally in spite of the fact that no functional pollen is produced. Somatogensis in F_1 hybrids of *Tabacum* with *sylvestris* seems, therefore, to be dominated by the *Tabacum* system as a unit, so that any particular modification of the *Tabacum* reaction system displays its full possibilities in the development of such hybrids.

Now, if these F_1 hybrids of *Tabacum* with *sylvestris* represent the reaction end-product of two fundamentally dissimilar reaction systems, then the relations of these two systems, as manifested by the domination of the *Tabacum* system to nearly or quite the exclusion of the *sylvestris* system, indicate a rather extensive mutual incompatibility of the elements of the two systems. This deduction is borne out by the fact that the F_1 *Tabacum-sylvestris* hybrids produce only a very few

functional ovules, the number of which is apparently constant within rather narrow limits. Assuming that segregation and recombination take place normally and in accordance with the chromosome view of heredity, these functional ovules represent the *Tabacum* and *sylvestris* extremes of a recombination series, the vast majority of the members of which fail to function because they are built up from incompatible elements derived from both systems. The evidence for such a constitution of the functional ovules is furnished by the results of backcrosses of the hybrid with the parents. When the F_1 hybrid is crossed back with *sylvestris*, the progeny consists of abnormal, sterile individuals and a few typical *sylvestris* individuals which are completely fertile and breed true. On the other hand when the *Tabacum* parent is crossed back onto the F_1 hybrid the progeny consists of *Tabacum* forms some of which are completely fertile and others of which are sterile like the F_1 hybrids. The hereditary phenomena, therefore, displayed by these F_1 species hybrids confirm the conception that they represent a contrast between reaction systems, the elements of which display a considerable degree of mutual incompatibility. It follows, then, that the type of behavior displayed by species hybrids may be considered as dependent upon the degree of incompatibility of the elements of the reaction systems therein involved. Sterility in such cases is merely a logical consequence of this same incompatibility, and the degree of sterility may be regarded as an expression of its extent.

The adoption and application of such a reaction system conception to hereditary phenomena has far reaching consequences. When, for example, this conception is applied to the *Oenothera* phenomena, it at once follows that the widespread occurrence of partial sterility, the significance of which has never been definitely determined, must be of primary importance in the formulation of any consistent explanation of the hereditary phenomena displayed in *Oenothera*. Until it is possible to define clearly the exact significance of this partial sterility, it is obviously useless to attempt to apply any rigid Mendelian analysis. Moreover, the *Oenothera* phenomena belong to several different categories, three of which, at least, may be clearly distinguished. In the first place, there are some strict factor mutations in *Oenothera*, such as *rubricalyx*. These mutations depend upon a particular change in some locus in the hereditary mechanism, and they display normal Mendelian behavior when tested with the forms from which they were derived. The extensive observations of Morgan and his associates on mutation in *Drosophila*, to say nothing of other well authenticated cases in both plants and animals, seem to establish the validity and nature of this

type of mutation beyond cavil or doubt. In the second place, there is a considerable series of forms which depend upon duplication of one or more or even all of the chromosomes to the extent of tetraploidy in some forms. The particular type of behavior displayed by such forms appears to depend upon changes in the proportions of the elements within the reaction systems, rather than upon actual changes in germinal substance. In the third place, there is a complicated group of phenomena which appear to be best considered as due to complex segregation of a type analogous to that displayed in wide crosses. In contrast to the simple and definite behavior of factor mutants, the forms resulting from this segregation are often distinctly different throughout from the forms from which they arose, and when tested with them, they exhibit a complicated but orderly type of hereditary behavior. There are two facts which stand out prominently with respect to this behavior—first the mutations affect the total ontogenetic development of the individual and second they tend to recur in relatively constant ratios in certain races. The definite ratio relations in the production of 'mutant' forms, the peculiar but orderly behavior of the hybridization phenomena, and the universal occurrence of partial sterility make together a series of facts which seem at least as consistently explainable on the basis of substratum hybridity as on assumptions of general germinal change. If the conceptions applied above to the behavior of species hybrids be extended in a somewhat modified form to the *Oenothera* phenomena, the occurrence of the 'mutants' and their subsequent behavior in hybridization admit of logical arrangement and interpretation without any necessity for assumptions of extensive germinal changes.

The experimental data cited above were obtained from cultures made possible by a portion of the Adams' Fund allotted to the Department of Botany by the Department of Agriculture of the University of California. A more detailed statement of the general position here outlined has been prepared and will appear in the near future.

POINT SETS AND ALLIED CREMONA GROUPS (PART II)

By Arthur B. Coble

DEPARTMENT OF MATHEMATICS, JOHNS HOPKINS UNIVERSITY

Received by the Academy, March 13, 1916

In Part I of this account[1] the ordered set P_n^k of n discrete points in a projective space S_k was studied with particular reference to its invariants, its association with a set Q_n^{n-k-2}, and its mapping upon a space $\Sigma_{k(n-k-2)}$. In this space Σ there was induced by permutation of the

points of P_n^k a Cremona group $G_{n!}$ which for $n = 5, 6$ led to new versions of the solutions of the quintic and sextic equations.

In Part II the novel idea of the *congruence* of point sets is introduced. This notion can be defined at once for a space S_2. Two mutually ordered point sets P_n^2 and $P_n'^2$ are congruent under a ternary Cremona transformation C with $\rho \leqq n$ fundamental points (F = *points*) if ρ of the pairs comprise the F-points of C and C^{-1} and if the remaining $n-\rho$ pairs of the sets are pairs of ordinary corresponding points of C. Thus the number of types of congruence depends upon the number of types of C and the number of ways of ordering the two sets. There is a natural pairing of the ρ F-points of C with the ρ F-points of C^{-1} and this is utilized to state in quite compact form the conditions for congruence up to $n = 9$. These conditions imply when $\rho = n$ a construction for the two sets of F-points and when $\rho < n$ a construction for the transformation C. The importance of the notion of congruence is due to the fact that two sets congruent in some order to a third are congruent in some order to each other. Thus if all the sets $P_n'^2$ congruent in some order to a given set P_n^2 be mapped upon points P' of $\Sigma_{2(n-4)}$ they form a conjugate set of points under the operations of a Cremona group $G_{n,2}$ in $\Sigma_{2(n-4)}$ which contains the $G_{n!}$ of Part I as a subgroup.

This definition of congruence cannot be extended immediately to sets in S_k. Let us first define a *regular* Cremona transformation C in S_k to be one which can be generated as a product of projectivities and of inversions of the variables. The regular transformations constitute a *regular Cremona group in* S_k. They are determined by their F-points precisely as in S_2. It is now possible to define as above congruence of sets P_n and $P_n'^k$ under regular Cremona transformation in S_k and the group $G_{n,k}$ in $\Sigma_{k(n-k-2)}$.

The effect of a regular transformation on spreads in S_k of order x_0 with multiple points of orders $x_1, \ldots x_n$ at the points of P_n^k is represented by a linear transformation on these variables with integral coefficients. In this way a group $g_{n,k}$ is derived which is isomorphic with $G_{n,k}$. This group reveals the striking analogy between the general transformation in S_2 and the regular transformation in S_k.

All sets $P_n'^k$ congruent to a set P_n^k upon an elliptic norm-curve in S_k can be projected upon the same curve. The transition from P_n^k to $P_n'^k$ is then effected by a linear transformation on the elliptic parameters $u_1, \ldots, u_n$ with rational coefficients. In this way a group $e_{n,k}$ is derived which also is isomorphic with $G_{n,k}$. The group is useful in determining the cases in which congruence implies projectivity.

The close relation of associated sets P_n and Q_n^{n-k-2} is again apparent

in that the groups $G_{n,k}$ and $G_{n,\,n-k-2}$ are identical while the groups $g_{n,k}$ and $g_{n,\,n-k-2}$, as well as the groups $e_{n,k}$ and $e_{n,\,n-k-2}$, are linear transforms of each other.

The group $G_{n,k}$ is in general infinite and discontinuous. The only finite types are the $G_{6,2}$, the $G_{7,2} = G_{7,3}$ and the $G_{8,2} = G_{8,4}$. These are identified with the well known groups of respectively the lines on a cubic surface, the bitangents of a quartic, and the tritangent planes of a sextic of genus 4 on a quadric cone. For these cases $G_{n,k}$ is in immediate algebraic relation to the corresponding geometric configuration. This advantage is used in the case of P_7^2 to determine the simplest system of irrational invariants of the point set which are invariants of the allied quartic as well. A similar method will be employed in Part III to handle the P_6^2 and the allied cubic surface. The first cases of infinite order, the $G_{9,2} = G_{9,5}$ and the $G_{8,3}$ can be adequately discussed by means of the isomorphic group $e_{n,k}$ and their structure has been determined.

Some interesting by-products are obtained. By means of $g_{n,k}$ a determination of all types of regular Cremona transformations with a single symmetrical set of F-points is made. The new types thus found are a transformation in S_4 of order 49 with 8 F-points of order 30 and a transformation in S_{2k-1} of order $(2k^2-1)$ with $2(k+1)$ F-points of order $2k(k-1)$. Also the discussion of $e_{9,2}$ leads to a determination of the infinite number of types of ternary Cremona transformations with 9 or fewer F-points in terms of 8 independent integers. Theorems such as the following:—*A pencil of plane cubic curves can be transformed by ternary Cremona transformation into only* 960 *projectively distinct pencils of cubics*—are proved for special sets P_9^2. Similar facts can be derived for P_8^2. Furthermore the general methods developed can be employed in the problem of determining the finite groups of regular transformations in S_k. For if a point P in $\Sigma_{k(n-k-2)}$ is fixed under a certain subgroup of $G_{n,k}$ the corresponding set P_n^k in S_k defines a regular Cremona group in S_k isomorphic with the given subgroup.

[1] These PROCEEDINGS, **1**, 245 (1915) and *Trans. Amer. Math. Soc.*, **16**, 155 (1915). This investigation has been carried out under the auspices of the Carnegie Institution of Washington, D. C.

ON A THEOREM OF LUCAS

By M. B. Porter

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TEXAS

Received by the Academy, March 10, 1916

In a paper on the *Geometry of Polynomials*[1] Lucas has an interesting generalization of Rolle's theorem, to wit: *That the zeros of any polynomial $F'(z)$ lie inside any closed convex contour inside of which the zeros of $F(z)$ lie.*

Many proofs[2] of this theorem have been given, but no one seems to have pointed out that the theorem is applicable to integral transcendental functions of the type $I_0(z) = \Pi_1^\infty (1-z/\alpha_1)$ where $\Sigma_1^\infty |1/\alpha_i|$ is convergent, i.e., functions of zeroth order (*genre* zero).

We shall show that this theorem can be generalized so as to give information concerning the distribution of the zeros of the derivative of certain rational functions and certain transcendental functions of the type $I_0(z)/\bar{I}_0(z)$.

We begin by giving a very elementary proof (perhaps new) of Lucas' theorem.

Proof. Since, in the finite part of the complex plane $F'(z)/F(z) = \Sigma_1^n(z-\alpha_i)^{-1}$, where $\alpha_1, \ldots, \alpha_n$ are the zeros of $F(z)$, can vanish only when $F'(z)$ vanishes, we have only to show that $\Sigma_1^n (z-\alpha_i)^{-1}$ can vanish only *inside* the convex contour mentioned. Now since the contour is convex, all the vectors $z-\alpha_i$ drawn from a point z *outside* the contour lie *inside* the arms of an angle *less* than 180°; the same thing will be true of the vectors $(z-\alpha_i)^{-1}$ (obtained by inverting and reflecting in the axis of reals through the point z). But such a set of vectors cannot form a closed[3] polygon, and hence the theorem is proved. It is now at once evident that the theorem is true for functions of the type $I_0(z)$ and we have, for example, a theorem of Laguerre's that: *If the zeros of $I_0(z)$ are all real, so are those of $I'_0(z)$.*

If $F(z) = P_n(z) / P_m(z)$, where P_n and P_m are polynomials or functions of the type $I_0(z)$ whose zeros α_i and β_i respectively lie inside of closed convex contours C_n and C_m which are *external* to each other, the proof given above shows that, if $\Phi(z) = P_n(z)/P_m(z)$, then

$$\frac{\Phi'(z)}{\Phi(z)} = \left(\log \frac{P_n(z)}{P_m(z)}\right)' \equiv \sum_1^n \frac{1}{z-\alpha_i} - \sum_1^m \frac{1}{z-\beta_i}$$

can have no zeros in the region swept out by such tangent straight lines to C_m as can be moved parallel to themselves into tangency with

C_n without cutting either C_m or C_n or passing through the position of the line infinity. This is a region which can be easily marked out on the complex plane and will have inside of it neither of the contours C_m and C_n.

Thus our theorem asserts that if the zeros of P_n are inside of one branch of an hyperbola and the zeros of P_m are inside the other branch, all the zeros of Φ' are *inside* of the hyperbola, or again, if all the zeros of P_n are real and lie in the interval (1) $x > a$, while all the zeros of P_m are real and lie in the interval (2) $x < b \leqq a$, then $\Phi'(z)$ has no complex zeros and all of its zeros lie in the intervals (1) and (2).

[1] *J. Ec. Polytech., Paris*, 28.

[2] All these proofs save one by Hayashi (*Annals of Mathematics*, March, 1914) are based on dynamical considerations. Fejér, Ueber die Wurzel vom kleinstein absoluten Betrage, etc., *Leipzig, Math. Ann.*, 65, 417, attributes the theorem to Gauss and gives a bibliography for it.

[3] If this is not at once intuitionally evident it can be shown by resolving the vectors in question into components parallel to the arms of the angle above mentioned.

INTERPRETATION OF THE SIMPLEST INTEGRAL INVARIANT OF PROJECTIVE GEOMETRY

By E. J. Wilczynski

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CHICAGO

Received by the Academy, March 13, 1916

If $y = f(x)$ is the cartesian equation of a plane curve, the integral

$$s = \int_{x_0}^{x_1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx, \tag{1}$$

which represents the length of the arc of this curve between the points $P_0(x_0, y_0)$ and $P_1(x_1, y_1)$, obviously remains unchanged when the curve is subjected to a plane motion. Therefore we may speak of s as an integral invariant of the group of motions, or as a *metric integral invariant*.

In the present paper we shall show how to find integrals connected with a given plane curve, whose values are not changed when the points of the plane are subjected to an arbitrary projective transformation. We shall speak of these integrals as *projective integral invariants*.

Let y_1, y_2, y_3, be the homogeneous coördinates of a point P_y, and let y_1, y_2, y_3 be given as linearly independent analytic functions of a parameter x. As x changes P_y will describe a non-rectilinear analytic curve C_y. There exists a uniquely determined linear homogeneous differential equation of the third order

$$y''' + 3p_1y'' + 3p_2y' + p_3y = 0 \tag{2}$$

of which y_1, y_2, y_3 form a fundamental system of solutions, and of which C_y shall be said to be an integral curve. All other integral curves of (2), associated with different fundamental systems of solutions, are projective transforms of C_y.

Since the coordinates y_1, y_2, y_3 are homogeneous, only those combinations of the coefficients p_1, p_2, p_3 can be of interest for the geometry of the curve C_y which depend only upon the ratios $y_1 : y_2 : y_3$. These combinations, the so-called *seminvariants* of (2), are all expressible as functions of

$$P_2 = p_2 - p_1^2 - p_1', \quad P_3 = p_3 - 3p_1p_2 + 2p_1^3 - p_1'' \tag{3}$$

and of their derivatives. The seminvariants of (2) are not altered if (2) is transformed by putting $\bar{y} = \lambda(x)y$ where $\lambda(x)$ is an arbitrary function of x.[1]

Although the seminvariants depend only upon the ratios $y_1 : y_2 : y_3$, they are still not adequate to represent the purely geometric properties of the curve C_y. The values of P_2 and P_3 depend also upon the special parametric representation which has been chosen for C_y. We may change this parametric representation in the most general way by putting $\bar{x} = \xi(x)$, where $\xi(x)$ is an arbitrary function of x. Those combinations of the seminvariants, called *absolute projective differential invariants*, which are left unaltered by all possible transformations of this sort, express intrinsic properties of the curve C_y. Moreover these properties are projective properties, since any projective transform of C_y may be regarded as an integral curve of (2).

Every absolute projective differential invariant can be expressed as a quotient of two relative invariants. The simplest of these relative invariants is[2]

$$\theta_3 = P_3 - \frac{3}{2}P_2'. \tag{3}$$

The property of θ_3 which justifies us in speaking of it as a relative invariant, is the following. Let us transform (2) by putting

$$\bar{x} = \xi(x), \quad \bar{y} = \lambda(x)y, \tag{4}$$

where $\xi(x)$ and $\lambda(x)$ are arbitrary functions of x. From the coefficients of the resulting differential equation between $\bar{x}$ and $\bar{y}$ let us form the quantity $\theta_3(x)$ according to the same rule which was used in forming θ_3 from the coefficients of (2). We shall find[2]

$$\bar{\theta}_3(\bar{x}) = \frac{\theta_3(x)}{(\xi')^3}. \tag{4}$$

This equation may be written

$$\bar{\theta}_3(\bar{x})\ (d\bar{x})^3 = \theta_3(x)dx^3.$$

Consequently the integral

$$p = \int_a^b \sqrt[3]{\theta_3\ (x)}\, dx \tag{5}$$

will not change its value under the transformations (4). Thus we see that the integral p is intrinsically connected with some geometric property of that arc of the curve C_y which corresponds to the interval $a \leqq x \leqq b$. It is also clear that this integral and its geometric significance will remain unaltered by any projective transformation of the plane, since it is expressed entirely in terms of the coefficients p_1, p_2, p_3 of (2) which are invariants of the projective group.

Therefore *the integral p, defined by (5), is a projective integral invariant.* If I is any absolute differential invariant of the curve C_y, the integral $\int Idp$ is again an integral invariant, and all integral invariants are expressible in terms of those obtained in this way.

We wish to explain the geometrical significance of the invariant integral p. For this purpose we need one further preliminary notion, namely that of the eight-pointic nodal cubic of a given point of a given curve.

A cubic curve is in general determined by nine of its points. If eight points only are given, there exist infinitely many cubics, forming a pencil, which pass through these points. In particular there exists a pencil of cubics, such that each cubic of the pencil has eight-pointic or seventh-order contact with the given curve C_y at a specified non-singular point P_y. One and only one of the cubics of this pencil has P_y, the point of contact, as double point. We call this cubic the *eight-pointic nodal cubic* of the point P_y, or the *penosculating nodal cubic* of P_y.[4]

The significance of the integral p is contained in the following theorem which we shall state without proof, but all of the terms of which have now been explained.

Consider an arc of an analytic curve corresponding to the interval $a \leqq x \leqq b$ of the independent variable. Divide this interval into n parts by means of the values $x_0 = a, x_1, x_2, \ldots x_{n-1}, x_n = b$, such that $\lim \delta x_k = \lim (x_{k+1} - x_k) = 0$ as n grows beyond bound. Let $A, P_1, P_2, \ldots P_{n-1}, B$ be the points on the curve which correspond to these $n + 1$ values of x. Let t_k be the tangent and C_k the eight-pointic nodal cubic of P_k. The three points of inflection of the cubic C_k are on a line i_k which interesects t_k in a point I_k. Denote by t_k one of the three inflectional tangents of C_k and let T_k be its intersection with

t_k. The line P_kP_{k+1} will intersect i_k and t_k in two points, I_k' and T_k', and the cross-ratio $(I'_k, T_k', P_k, P_{k+1})$ turns out to be equal to

$$1 - \frac{3}{\sqrt[3]{10}} \sqrt[3]{\theta_3(x_k)}\, \delta x_k \tag{6}$$

except for terms of higher than the first order in δx_k.

By a perspective correspondence the three points I'_{n-1}, T'_{n-1}, P_{n-1} of $P_{n-1}B$ may be projected into the points $I'_{n-2}T'_{n-2}$, P_{n-1} of $P_{n-2}P_{n-1}$. Let B_{n-1} be the point of $P_{n-2}P_{n-1}$ which, in this perspective, corresponds to B. Then project similarly I'_{n-2}, T'_{n-2}, P_{n-2}, B_{n-1} into the four points I'_{n-3}, T'_{n-3}, P_{n-2}, B_{n-2} of $P_{n-3}P_{n-2}$, and continue in this way. We shall finally obtain upon the line AP_1 a point B_1 determined from B by this sequence of perspectives. As n grows beyond bound, B_1 will approach a limiting position Q on the initial tangent t_0 of the arc AB. The cross-ratio

$$k = (I_0, T_0, A, Q) \tag{7}$$

will be the limit which the product

$$\prod_{k=1}^{n} \left(1 - \frac{3}{\sqrt[3]{10}} \sqrt[3]{\theta_3(x_k)}\, \delta x_k\right) \tag{8}$$

approaches when n grows beyond bound. Consequently we find

$$\log k = \frac{3}{\sqrt[3]{10}} \int_a^b \sqrt[3]{\theta_3(x)}\, dx. \tag{9}$$

This equation contains the desired interpretation of the integral p.

From a theoretical point of view the expression (5) for the integral p is the simplest and most general. We shall however give, in conclusion, three other expressions for p in terms of more familiar variables.

If the curve is given by means of its cartesian equation in the form $y = f(x)$, we may write

$$p = -\frac{1}{3\sqrt[3]{2}} \int \frac{\sqrt[3]{9(y'')^2 y^{(5)} - 45\, y'' y''' y^{(4)} + 40\, y^{(3)3}}}{y''\sqrt{1+y'^2}}\, dx, \tag{10}$$

where $y' = dy/dx$, $y'' = d^2y/dx^2$, and so on.

If the curve is given by means of parametric equations of the form $x = \varphi(s)$, $y = \psi(s)$, where s denotes the length of arc, and if r is the radius of curvature at the point which corresponds to the value s of the parameter, we find

$$p = \frac{1}{\sqrt[3]{6}} \int \sqrt[3]{\frac{r'''}{r} - \frac{r'r''}{r^2} + \frac{4}{9}\frac{r'^3}{r^3} + 4\frac{r'}{r^3}}\, ds, \tag{11}$$

where $r' = dr/ds$, $r'' = d^2r/ds^2$, etc..

Finally we may write

$$p = \int \frac{1}{\rho_0} \sqrt[3]{\frac{r_0}{2r}}\, ds, \qquad (12)$$

where r has the same meaning as in (11), where ρ_0 is the distance from the point P of the curve to the center M of the corresponding osculating conic, and where r_0 is the radius of curvature at M of the locus which M describes when P moves along the given curve.

[1]E. J. Wilczynski, *Projective Differential Geometry of Curves and Ruled Surfaces*, p. 58. [2]Loc. cit., p. 59. [3]Loc. cit., p. 60. [4]Loc. cit., pp. 67-68.

SIZE INHERITANCE IN GUINEA-PIG CROSSES

By W. E. Castle

BUSSEY INSTITUTION, HARVARD UNIVERSITY

Received by the Academy, March 9, 1916

For several years my pupils and I have been engaged in studying the inheritance of size and weight differences among animals, these being characteristics of much economic importance and of peculiar theoretical interest. Preliminary studies published in 1909 showed that size and weight in rabbits do not follow the Mendelian rules of dominance and segregation as unit-characters. But Lang subsequently suggested that multiple Mendelian factors may be concerned in such cases, extending to animals a principle already recognized by Nilsson-Ehle in dealing with certain categories of characters in plants. Punnett and Bailey (1914) accept this principle in explaining weight inheritance in crosses of bantam fowls with those of ordinary size. They believe that four differential factors are concerned in a particular cross studied, three dominant factors which tend to increase size being found in the larger race, one such factor being found in the bantam race. By recombination in F_2 some individuals are obtained smaller than the bantam race, and others in F_2 larger than the larger race. But there are some reasons for questioning the validity of this analysis which assigns very definite quantitative values to the several hypothetical factors, without however making any allowance for physiological changes of size due to non-genetic causes, or for possible quantitative variation in the factors themselves. Moreover, let it be granted for the sake of argument that these four Mendelizing factors exist and that each is an independent agency for increasing size. On the Mendelian hypothesis there should be obtained from the cross in question individuals which lack *all four* of these factors. What, it may be asked, will their size be? Will they be with-

out size or devoid of genetic variation in size? No race of animals has yet been shown to exist which is devoid of genetic variation in size,—Jennings' supposed demonstration of such a condition in asexually reproducing Paramecium having been disproved by the work of Calkins and Gregory and his own subsequent work on Difflugia. If when all discoverable Mendelian factors are wanting size still varies genetically, a Mendelian explanation of size inheritance proves itself inadequate. There also exists a possibiity that *bantamness* in fowls involves discontinuous variation such as does not occur in ordinary size differences. For Phillip's results with ducks do not show such segregation of the extremes of size as Punnett and Bailey record, though his crosses were made between races of ducks both very pure and very different from each other in average size. What Phillips observed was a blend with a slight increase of variability in F_2 but without evidence of complete segregation or recombination of size factors.

Some excellent material for the study of size inheritance has become available to the writer in certain very pure races of guinea-pigs differing widely as to size and it is the purpose of this note to describe briefly the more important results obtained and their possible significance. Attention is invited to the nature of the growth curves observed for the races crossed and to non-genetic as well as to genetic factors affecting size.

Three distinct and unrelated races were used in the crosses to be described. First, a wild race obtained at Arequipa Peru in December 1911, and identified as Cavia Cutleri Bennet. This has been bred for three generations in captivity at the Bussey Institution and has shown itself very uniform in size and other characters. Second, a race of guinea-pigs which we may call race B, bred distinct for many years at the Bussey Institution, very uniform in size, and varying as regards Mendelian factors only in respect to the color factor. Some animals of this race are black, others are albino. Both sorts are alike as regards other characters. A race C, also long bred distinct at the Bussey Institution, is of about the same average size as race B. It as well as race B has been used in crosses with C. Cutleri, but the hybrids from these two crosses have not been interbred, though the data concerning them have been combined for statistical treatment.

Figure 1 shows separately growth curves for the two sexes of race B and Cavia Cutleri. These curves have been obtained by combining the individual growth curves of several different animals reared in captivity and weighed at intervals of one or two weeks. They represent *averages* and the curves have been smoothed somewhat. It will be observed that the young of Cutleri grow rapidly for the first 40 or 50 days of their

lives after which increase in weight slows up gradually and finally ceases altogether at the age of about one year. Both sexes are of about the same size at birth but females grow at first faster than males, attaining sexual maturity some weeks earlier. At about fifty days of age the sexes are again of the same average size, but from this time on the males are heavier. The general form of the growth curve in both sexes is, as in animals generally, at first concave upward but later becomes convex upward as the growth rate declines. Growth is completed at about one year, after which time no change in weight occurs except as caused by health or food conditions, or pregnancy of the females. The growth curve of C. Cutleri may be characterized as a sharply rising, flat curve. Compared with this the growth curve of the domesticated race B rises less rapidly at first but continues its rise longer. The same difference

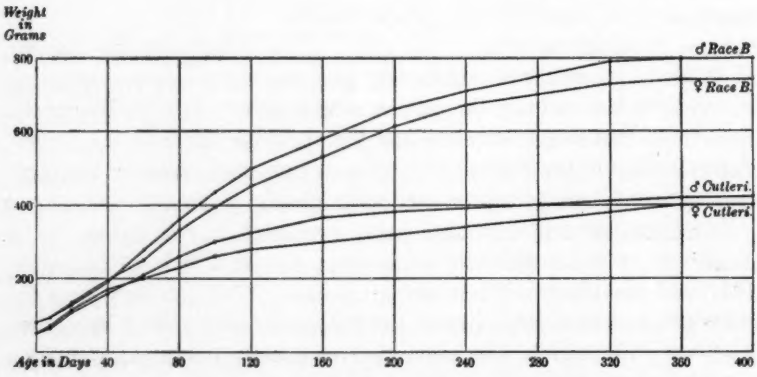


FIG. 1. GROWTH CURVES OF RACE B GUINEA-PIGS AND OF CAVIA CUTLERI.

in the growth rate of the sexes is observed in race B as in the wild C. Cutleri. Females grow faster up to 40 or 50 days of age, after which time males permanently take the lead. The adult weight of race B animals is approxmately double that of Cutleri individuals.

In figure 2 are reproduced for comparison the growth curves of the females of both race B and Cutleri and along with these are plotted the growth curves of F_1 and of F_2 females produced by crossing Cutleri males with females of race B and race C. These curves show that F_1 females are larger at birth than females of either parent race and that they remain continuously larger having at maturity a size slightly larger than that of either parent race. That this increased size is not due to heredity, but to a growth stimulus produced by the crossing of unrelated parent races, a 'law of hybridization' formulated by Focke (1881), is shown by

an examination of the F_2 growth curve, which is everywhere lower than the F_1 growth curve. The F_2 females are indeed, like F_1 females, at first larger than young of either pure race and continue this lead for three or four months, but from that time on their growth rate slows up notably and falls below that of race B so that at maturity they are nearly intermediate in size between the parent races. The F_2 females, as a group, have a rapid early growth like that of the Cutleri parent but lack the staying qualities of race B as regards growth so that from an age of about 150 days on they fall below race B in size. Accordingly, though the growth curves of both parent races and of the F_1 females are so far parallel in general course that they do not cross each other, the growth curve of the F_2 females crosses and falls below that of race B attaining at length an intermediate position between Cutleri and race B. This, so far as heredity is concerned, we may assume to be its true position,

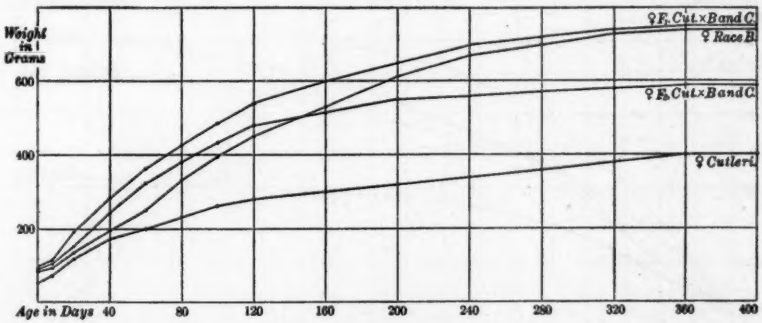


FIG. 2. GROWTH CURVES OF RACE B AND CUTLERI FEMALES AND OF THEIR FEMALE HYBRIDS BOTH F_1 AND F_2.

the added size of F_1 females being due not to heredity but to a growth impetus arising from hybridization, in accordance with Focke's law.

Figure 3 shows that the statements made concerning female hybrids are true also for male hybrids in crosses between Cutleri males and females of race B or race C. In this case the superior size of F_1 hybrids, as compared with the parent races, is even more strongly in evidence, while F_2 hybrids, though larger than either parent race for about four months, from that time on are of intermediate size. But for some reason, possibly not due to heredity, their ultimate size is nearer to that of race B than to that of the Cutleri parent.

It might be supposed that the size inheritance in crosses of a wild species with a domesticated race may be different from that in crosses between different domesticated races. To test this matter crosses were made between females of race B and a male guinea-pig obtained in the

cabin of a native in Arequipa, Peru, this Arequipa guinea-pig being of a size about one half greater than that of race B males. The growth curve of race B males is shown in figure 4 where it can be compared with the growth curves of F_1 and F_2 male animals obtained by crosses of the Arequipa male with race B females. It will be observed that the growth curve of the F_1 males runs a course in general parallel with that of the growth curve for race B males but at a much higher level. The growth curve of the F_2 males starts in at a still higher level, perhaps in part because the vigorous F_1 mothers supply an abundance of nourishment to the young. But from weaning time on (age about 20–30 days) the F_2 males grow less rapidly. Their curve crosses the F_1 curve at 40 days and continues thence below it so that its ultimate position is

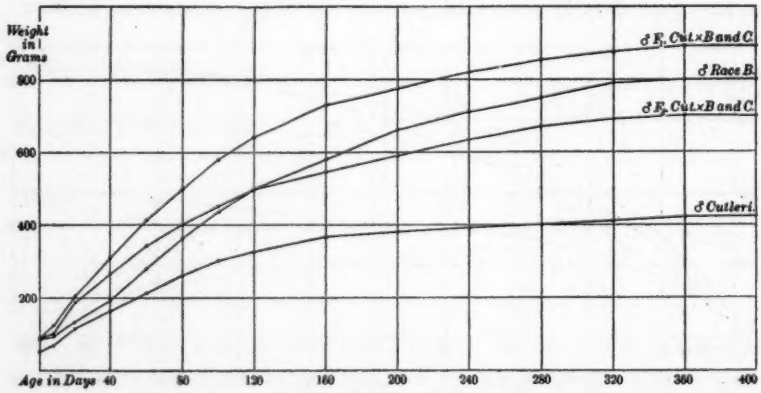


FIG. 3. GROWTH CURVES OF RACE B AND CUTLERI MALES AND OF THEIR MALE HYBRIDS BOTH F_1 AND F_2.

intermediate between the F_1 and the race B growth curves. It is evident that here, as in the Cutleri crosses, the F_1 animals are larger than heredity alone would make them, that they have an added size due to a growth stimulus produced by hybridization, but that this added stimulus is largely dissipated in F_2, which accordingly shows blended or intermediate heredity. It is evident that Mendelian dominance is wanting in these size crosses. It remains to inquire whether segregation also is wanting. The individual growth curves of F_1 and F_2 animals give no very clear evidence on this point. Too many environmental factors enter into the problem, such as time of year, food conditions, accumulation of fat in old age, and the like, which, while they do not affect the *average* growth curves already discussed, do obscure the question of

segregation in individual cases. Bone measurements are believed to be less affected by extraneous influences than the growth curves of individuals. Consequently our inquiry as to evidences of segregation will be confined to them. Three different bone measurements have been studied for each full grown individual (age 13 months or over, in most cases). These are maximum skull-length, maximum skull-width, and femur length. The facts arrived at are contained in Tables 1–3.

In the first part of Table 1 is shown the variation in skull-length of 10 full grown Cutleri females and of 28 full grown race B females. Measurements were made with a caliper rule furnished with a vernier,

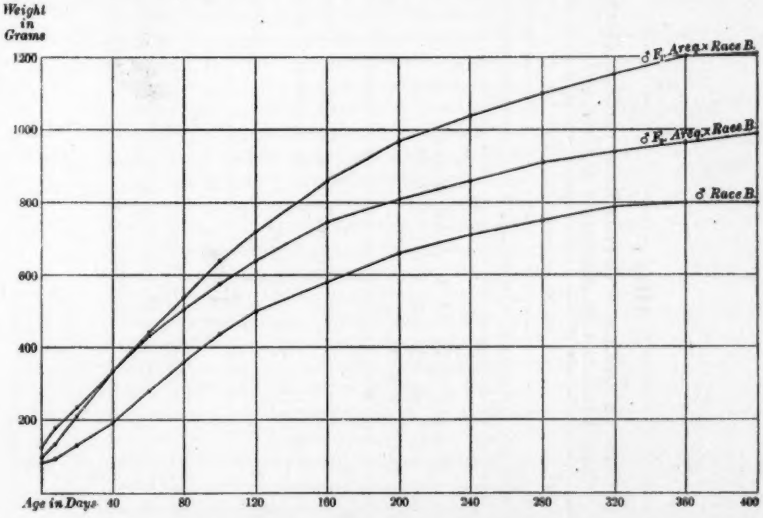


FIG. 4. GROWTH CURVES OF RACE B MALES AND OF THE MALE HYBRIDS BOTH F_1 AND F_2, PRODUCED BY A CROSS WITH THE AREQUIPA RACE.

and are accurate within 0.1 mm., the possible observational error shown by repeated measurement. The measurements are classified in classes of 0.5 mm. range numbered in order of increasing size. The lowest class, that numbered 1, would include measurements 48.5–48.9 mm.; class 2 would include measurements 49.0–49.4, etc. The absolute measurements included within each class of the table may thus be readily calculated, if desired. They are omitted for simplicity. The Cutleri females measured range in skull-length from class 1 to class 10. The race B females range from class 11 to class 25. Thus the two races do not overlap in range. Race B begins where Cutleri leaves off. Their means are 6.6 mm. apart.

TABLE 1

VARIATION IN SKULL-LENGTH OF CAVIA CUTLERI AND OF CERTAIN RACES OF GUINEA-PIGS AND OF THEIR HYBRIDS
CLASS 1 INCLUDES MEASUREMENTS 48.5–48.9 MM.

	0.5 MM. CLASSES																																TOTALS	MEANS	σ
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38			
♀ C. Cutleri.....	1			2			2	2	2	1																													10	51.55	13.50
♀ Race B.....											1		1	2	2	1		2	1	3	3	4	3	1	4													28	58.14	19.75
♀ F_1 Cutleri × Race B												1		4		2		1	3	4	2	4	2	1														24	57.70	16.85
♀ F_2 Cutleri × Race B	1						1	3	2	1	1	6	7	4	1	2	3	1																				33	54.35	17.20
♂ C. Cutleri.....							2		2		2	1																										7	52.91	9.45
♂ Race B.....																		2		3	6	7	9	9	7	4	7	4	1	2	1	1						63	60.35	15.05
♂ F_1 Cutleri × Race B																				1		1	3	2	7	5	2	2	1	1	1							26	61.20	12.15
♂ F_2 Cutleri × Race B										1		1	1	1	2	2	3		3	4	4	1								1									24	57.26	20.00
♀ F_1 Areq. × Race B.																							1		3	4	3		2	2	3							18	61.92	12.10
♀ F_2 Areq. × Race B.															1		1		2	4	3	4	4	2	1	3	5	2	4	3	1	1						41	60.44	20.65
♂ F_1 Areq. × Race B.																										1	1		3	2	3	5	1	4	4		1	2	27	64.40	15.15
♂ F_2 Areq. × Race B.																		1				2	7	5	3	3	9	5	5	4	2	3	1	1				56	61.86	17.75

TABLE 2

VARIATION IN SKULL WIDTH OF CAVIA CUTLERI AND OF CERTAIN RACES OF GUINEA-PIGS AND OF THEIR HYBRIDS
CLASS 1 INCLUDES MEASUREMENTS 29.5–29.9 MM.

	0.5 MM. CLASSES																									TOTALS	MEANS	σ
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27			
♀ C. Cutleri	2	2	2	1	2		1																					10	30.84	9.35
♀ Race B							1	3	4	4	3	8	1	2	2													28	34.68	10.56
♀ F_1 Cutleri × Race B							1		3	1	5	3	6	1	2	1	1											24	35.24	11.60
♀ F_2 Cutleri × Race B			1	1	1	1	8	9	5	2	3			2														33	33.26	11.45
♂ C. Cutleri			2		3	1	1																					7	31.63	6.80
♂ Race B									1	2	4	6	11	13	11	2	8	1	2	2								63	36.33	11.90
♂ F_1 Cutleri × Race B													1	4	2	2	5	5	2	3	1	1						26	37.79	11.70
♂ F_2 Cutleri × Race B							1		2	1	9	1	5	1	1	1	2											24	35.24	12.05
♀ F_1 Areq. × Race B															2	2	1	3	6	1	3							18	38.37	9.40
♀ F_2 Areq. × Race B												2	2	10	7	5	1	5	1	3	4		1					41	37.35	14.05
♂ F_1 Areq. × Race B																	5	1	5	2	5	3	3	2	1			27	39.40	11.95
♂ F_2 Areq. × Race B																3	10	7	17	5	4	4	3	2			1	56	38.87	11.40

TABLE 3

VARIATION IN FEMUR-LENGTH OF CAVIA CUTLERI AND OF CERTAIN RACES OF GUINEA-PIGS AND OF THEIR HYBRIDS
CLASS 1 INCLUDES MEASUREMENTS 35.5–35.9 MM.

	0.5 MM. CLASSES																							TOTALS	MEANS	σ
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24			
♀ C. Cutleri		1	1	1	1	1	1	3	3																12	38.45	12.05
♀ Race B							3		2	3	2	6	5	5	1	1	1								29	41.16	12.50
♀ F_1 Cutleri × Race B											2	2	2	4	2	4	6	1							23	42.63	10.35
♀ F_2 Cutleri × Race B	1					2	1	5	3	6	2	6	3		3		1								33	40.38	15.60
♂ C. Cutleri					2		2	2		1															7	38.77	8.20
♂ Race B									1		2	6	9	10	12	4	3	3	1	1					52	42.39	10.70
♂ F_1 Cutleri × Race B											1				4	2	5	3	4						19	43.57	9.80
♂ F_2 Cutleri × Race B								2	1	2	5	4	6		2	1						1			24	41.32	14.20
♀ F_1 Areq. × Race B														3	3	3	1	4	4	1	2		2		23	44.07	13.35
♀ F_2 Areq. × Race B										2	1	5	3	6	4	5	3	5	3	1	2	1			41	42.95	15.30
♂ F_1 Areq. × Race B																2	1	5		5	8	2	1	1	25	45.16	10.05
♂ F_2 Areq. × Race B										1	1	1	4	9	8	7	10	6	5	1			1		54	43.15	11.80

F_1 female hybrids between Cutleri and race B are practically as large in skull-length as race B females. Their means differ by less than half a millimeter. This harmonizes with the observations on general size as indicated by the weights given in the growth curves. The empirical range of the F_1 hybrids is from class 12 to class 24, just within the limits of variation of race B. The F_2 hybrids are considerably lower in mean skull-length than their F_1 parents: indeed they are strictly intermediate between pure Cutleri and race B, in complete agreement with the growth curves (fig. 2). Their mean skull-length is 54.35 mm.; the middle point betwen the skull-length of Cutleri and race B is just 0.5 mm. greater. Their range (if we leave out of consideration one aberrant individual) extends from the middle of the range of Cutleri to the middle of the range of race B, and would show no increase of variability over F_1 or race B. But the one aberrant individual makes F_2 more variable than F_1; it is as small as the smallest Cutleri female in skull-length and also in femur length (Table 3). It was however not so small in skull-width (Table 2). Nevertheless it might pass for a very good size segregate closely resembling the Cutleri ancestor. There can be no doubt about its hybrid origin or that it was a genuine F_2, not an accidental back-cross of F_1 with Cutleri, for it was of a color variety, cinnamon, which could not be obtained from a back-cross or from either uncrossed parental race. We shall consider its significance further.

In the second section of Table 1 are classified the skull-length measurements of Cutleri and race B males and of male hybrids between these races, both F_1 and F_2. These show that the skull-length of males regularly exceeds that of females of like ancestry but that the relations of race to race, as regards skull-length, are the same for males as for females. Cutleri and race B do not overlap in range of skull-length in either sex. Their F_1 hybrids equal or exceed race B in mean skull-length, but show no greater variability. F_2 is intermediate in skull-length between the parental races, lacking the vigor of F_1 due to crossing but not a matter of heredity. There is however, among the male F_2 hybrids, one aberrant individual with a long skull comparable with that of the largest race B animals and Table 3 shows that one of the F_2 males (the same one in fact) even surpassed race B males in femur length. But in Table 2 we look in vain for an aberrant F_2 male of unusual skull-width. The animal in question accordingly had an unusually long but not an unusually wide skull. It also had an unusually long femur. Obviously some agency is at work which produces variation in *length* of skull and femur without greatly affecting width of skull. The aberrant F_2 female as well as the aberrant F_2 male indicate this. Whether the hypothetical

agency is genetic or purely physiological (like that which produces the large size of F_1) is at present uncertain. If it should prove to be genetic, it may conceivably be a Mendelizing factor of greater or less stability, like those which affect the shape of fruits in tomatoes, peppers, and squashes (Gross, Emerson). But this will remain a matter of uncertainty until further evidence is forthcoming. Aside from these two individuals with skeletons of unusual *form*, everything so far indicates that the inheritance of skeletal dimensions is completely blending with physiological increase of size in F_1, this being however not a matter of heredity.

We have already anticipated the result of the Cutleri × race B cross as regards skull-width. See Table 2. Here also the parent races scarcely overlap in range of variation. The mean of F_1 slightly exceeds that of race B. F_2 is intermediate between the parent races and scarcely more variable than F_1, with no very aberrant individuals.

In the lower portion of Tables 1–3 will be found the bone-measurements for the F_1 and F_2 generations produced by a cross between race B females and the Arequipa male, 1002, still living and so not available for bone measurements.

But bone measurements of a pair of animals of the Arequipa race are slightly less than the corresponding average measurements of the F_1 animals. Hence it is probable that a considerable physiological increase of size occurs in F_1 in this cross, as in the Cutleri cross, causing the production of an F_1 larger than either parent race. Comparison of the growth curves of F_1 and F_2 (figs. 3 and 4) and of their respective bone measurements confirms this idea. The mean of F_2 is in all measurements less than that of F_1, the difference amounting to one or two millimeters. This indicates that the increase of size due to crossing, seen in F_1, does not persist in F_2.

But the important question theoretically is whether there is evidence of the segregation or recombination in F_2 of distinct genetic size factors. If such segregation occurs, it might be expected to show itself either (1) in increased amplitude of the variation, provided that multiple factors occur which lack dominance but segregate and recombine independently of each other, or (2) in the formation of multi-modal F_2 variation curves with the production of isolated aberrant individuals, provided that the genetic factors concerned are few in number or show dominance.

As regards the first possibility, the F_2 animals do show somewhat greater variability than the F_1s, though the difference as measured by the standard deviation (σ) is not great. In the case of Cutleri × race

B females, the standard deviation of F_2 in skull-length (Table 1) is less than that of uncrossed race B females and is not half a millimeter greater than that of the F_1s. The difference can scarcely be regarded as significant. In skull-width the F_2 females have a smaller standard deviation than the F_1s, while in femur-length alone is F_2 larger than F_1 and either parent race.

Among the male hybrids produced by the Cutleri × race B cross, the evidence for increased variability in F_2 is rather better. In skull-length (Table 1) and femur-length (Table 3) the standard deviation of F_2 exceeds that of F_1 or either uncrossed race. In skull-width the difference is not significant.

It should be noted in passing that the very low standard deviations of pure Cutleri animals do not necessarily indicate lack of variability, but are to be explained in part as due to the small numbers of animals available for measurement. For Pearson has shown that with numbers less than 25, the empirical standard deviation is as a rule too small. It will be observed in Tables 1–3 that the highest standard deviations among the pure Cutleri animals are found where the numbers studied are largest. As between the F_1 and F_2 animals studied, the numbers are not sufficiently different to make any allowance of this sort necessary. In the Arequipa × race B crosses, F_2 has a higher standard deviation than F_1 in five out of six cases, the F_1 animals being more variable only in the skull-width measurements of males (Table 2).

We may conclude that on the whole F_2 shows consistently a higher variability than F_1 in the croses studied, which is in agreement with the observations made in numerous other hybridization experiments with both animals and plants. But it does not follow that the difference observed is due wholly to multiple genetic factors affecting size. For increased variability in F_2 would occur if the physiological increase of size observed in F_1 persisted to some extent in F_2 but persisted unequally (i.e., in different degrees) among the different F_2 zygotes. Now there is some reason to think that the non-genetic or excess vigor of F_1 does persist slightly into F_2, for in 5 out of 6 cases in the Cutleri × race B cross (Tables 1–3), the F_2 mean is *greater* than the intermediate point between the means of the uncrossed races. But it is clear that if this non-genetic vigor is found to a greater extent in some F_2 zygotes than in others, *it will increase the variability of the F_2 zygotes as a group.*

As against the multiple factor hypothesis it may be urged further that an increased variability of F_2 may be satisfactorily accounted for in still other ways without involving multiple factors, as for example by quantitative variation in a single factor affecting total growth energy of the zygote.

We come now to the alternative question whether evidence is forthcoming of a *few* genetic factors or of single genetic factors affecting size, showing themselves in multimodal variation or in aberrant individuals. There is in Tables 1–3 no evidence of multimodal variation. As regards aberrant individuals we have two and only two notable cases, which are found in F_2 of the Cutleri × race B cross, as already stated. These are (1) a female with very short skull and femur closely resembling pure Cutleri females in these measurements, but not in skull-width, and (2) a male with very long skull and femur, closely resembling race B animals in these respects but not in skull-width. It seems quite possible that some special factor affecting duration of growth is concerned in the production of these extreme individuals. Thus castration of cattle is known to permit abnormally prolonged growth of the skeleton *in length*. Early sexual maturity is probably correlated with an opposite change. Whether physiological factors affecting the sexual system are concerned in the production of these unusual individuals, or whether genetic agencies are concerned is at present uncertain. It is regrettable that the exceptional character of these two individuals was not recognized while they were still alive and so capable of genetic tests. Possibly further investigations now in progress may throw light on these questions.

If the foregoing reasoning is sound, we have from these crosses no evidence showing either the existence of *numerous* multiple Mendelian factors affecting size, or the existence of a *few* Mendelian factors affecting size, or the existence of a *single* Mendelian factor affecting size. We have however clear evidence of a physiological factor causing a marked increase of size in F_1 and probably persisting to a small extent in F_2 and so increasing slightly the variability of F_2. This factor, though its existence has long been known, has been largely neglected in those recent studies of size inheritance which seek to give a Mendelian interpretation of the phenomena observed in size crosses, and it is quite possible that this neglect has caused erroneous conclusions to be reached.

Castle, W. E., and H. E. Walter, R. C. Mullenix, and S. Cobb, Studies of inheritance in rabbits, Carnegie Inst. of Wash., *Publ.* No. 114 (1909).

Lang, A., Die Erblichkeitsverhältnisse der Ohrenlänge der Kaninchen nach Castle and das Problem der intermediaren Vererbung and Bildung konstanter Bastardrassen. *Zs. indukt. Abst. Vererbungs-lehre*, 4, 1–23. (1910.)

Nilsson-Ehle, H., Kreuzungsuntersuchungen an Hafer und Weizen, I. *Lunds Universitats Årsskrift*, N. F., 7 (1909).

Phillips, J. C., Inheritance of size in ducks. *J. Exp. Zool.*, 12, 193 (1911); A further study of size inheritance in ducks with observations on the sex ratio of hybrid birds. *Ibid.*, 16, 131–148 (1914).

Punnett, R. C., and P. G. Bailey, On inheritance of weight in poultry. *J. Genetics*, 4, 23–39, (1914) Pl. IV.